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A SIMPLE ANALYTICAL MODEL FOR DENSE WDM/OOK SYSTEMS

by

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June 1994

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A SIMPLE ANALYTICAL MODEL FOR DENSE WDM/OOK SYSTEMS

by

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ABSTRACT

We derive the closed form expression for the bit error probability of dense WDM systems employing an external OOK modulator. Our model is based upon a close approximation of the optical Fabry-Perot filter in the receiver as a single-pole RC filter for signals that are bandlimited to a frequency band approximately equal to one sixtieth of the Fabry-Perot filter's free spectral range. Our model can handle bit rates up to 2.5 Gb/s for a free spectral range of 3800 GHz and up to 5 Gb/s when the power penalty is 1 dB or less.

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I. INTRODUCTION

Wavelength division multiplexing (WDM) systems have been increasingly proposed as an attractive alternative to coherent optical frequency division multiplexing (FDM) systems [Ref. 1-5]. Although WDM systems with direct detection do not have the channel capacity of coherent optical FDM systems, they are much less costly to implement. Furthermore, present filter technology enables the designers to tightly pack the channel, resulting in dense WDM systems that can provide aggregate bit rates of many terabits per second (1 Tb/s = 10^{12} b/s). Dense WDM systems are particularly attractive in the area of undersea surveillance where hundreds of sensors and data collection sites are envisioned being merged onto single-fiber superhighways through massive data fusion. Other applications call for relatively low-bandwidth data collection over many months to be dumped quickly to a remote recording site in a matter of minutes. This "collection-and-dumped" compression can demand total data rates on the order of hundreds of Gb/s. Long distance between data collection sites and a remote recording site requires the use of optical amplifiers. Therefore, it is necessary to pack all channels within the optical amplifier bandwidth.

A dense WDM receiver with on-off-keying (OOK) modulation can be modeled as shown in Fig. 1. Conceptually, the analysis involves two main operations: 1) a convolution operation to evaluate the signal at the output of the optical filter, a

Fabry-Perot (FP) filter in our investigation, and 2) the integration of the output of the photodetector. Evaluation of bit error probability by the numerical analysis of these two operations has been carried out in [Ref. 6], with a number of approximations made to reduce the computational complexity. In this investigation the FP filter is shown to be well approximated by an RC filter within the frequency range $|f-f_0| < FSR / 20\pi$, where FSR is the free spectral range of the FP filter [Ref. 7] and f_0 is the FP filter center frequency. For example, given FSR = 3800 GHz, the approximation works very well for $| f-f_0 | < 60.5$ GHz; that is, the effects of adjacent channels within a 121 GHz bandwidth centered at f_0 must be included, while all others can be neglected. This simple model agrees well with [Ref. 6] as demonstrated in Section III. Furthermore, this model enables us to obtain a closed form analytical expression for the bit error probability for which numerical results can be obtained with little effort. Our investigation shows that this simple model provides accurate results as compared to those in [Ref. 6] for bit rates up to 2.5 Gb/s when the effects of four adjacent channels are included with FSR = 3800 GHz. Actually, when the power penalty relative to single channel operation is 1 dB or less, there is virtually no difference in the effect of four or two adjacent channels. Thus, for this power penalty criterion, this simple model can handle bit rates up to 5 Gb/s for a FP filter's FSR = 3800 GHz.

In Section II the closed form expression for the decision variable, and consequently, the bit error probability assuming all channels are bit synchronous as in [Ref. 6] is derived. Section III presents the numerical results which include the bit error probability versus the

signal-to-noise ratio as function of the FP filter bandwidth and channel spacing, and the power penalty (relative to single channel operation without filtering or with filtering but no intersymbol interference) versus the channel spacing as a function of the bandwidth. Finally, a summary of results appears in Section IV.

II. ANALYSIS

The receiver model for the dense WDM system is shown in Fig. 1. The desired signal is filtered by a Fabry-Perot (FP) filter that rejects adjacent channels. The photodetector is assumed to have a responsivity $\mathcal{R}$ (A/W). The detected current is amplified by a low noise amplifier that contributes a postdetection thermal noise $n(t)$ with spectral density N_0 (A^2/Hz). The decision variable at the output of the integration is compared to a threshold α to determine whether a bit zero or bit one was present.

A. INPUT SIGNAL

For convenience, we designate channel 0 as the desired channel, and channel k as an adjacent channel where $k = -M/2, \ldots, -1, 1, \ldots, M/2$ with M an even integer. We consider the equivalent lowpass (complex envelope) data signal in channel 0 and channel k as follows:

$$b_0(t) = \sum_{i=-L_0}^{0} b_{0,i}\, p_T(t - iT) \tag{1}$$

$$b_k(t) = \sum_{l=-L}^{0} b_{k,l}\, e^{j\omega_k t}\, p_T(t - lT) \tag{2}$$

where

T: bit duration

$b_{0,i} = \{0,1\}$: bit in channel 0 in the time interval $(iT, (i+1)T)$

$b_{k,l} = \{0, e^{j\phi_k}\}$ is the l^{th} bit in channel k in the time interval $(lT, (l+1)T)$

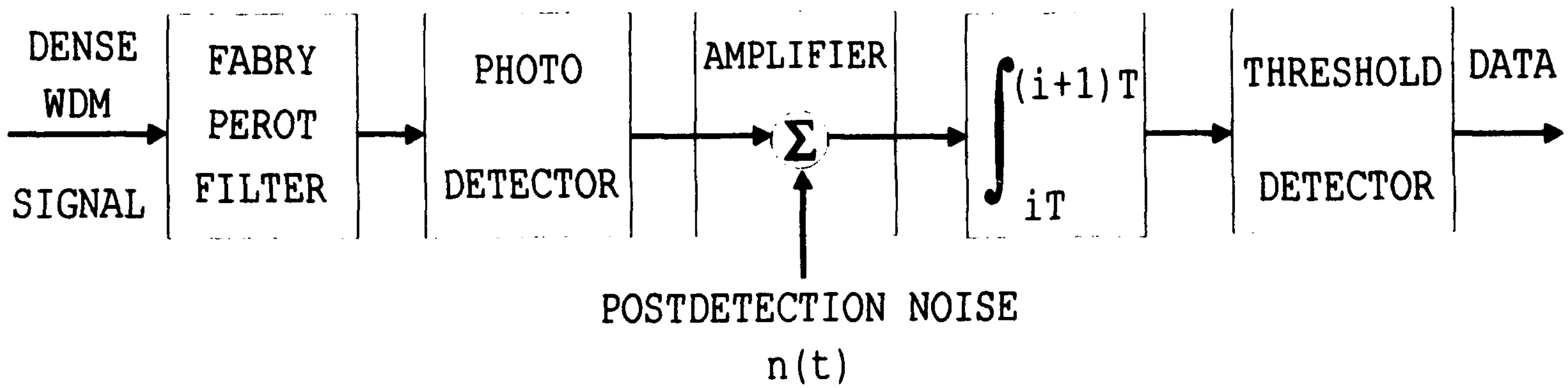


Figure 1: OOK receiver structure

ϕ_k : a phase offset between channel k and channel 0 and is assumed to be uniformly distributed in $(0, 2\pi)$ radians

ω_k : radian frequency spacing between channel k and channel 0 with $\omega_k = -\omega_{-k}$

The function $p_T(t - iT)$ is defined as

$$p_T(t-iT) = \begin{cases} 1, & iT<t<(i+1)T \\ 0, & \text{otherwise} \end{cases} \tag{3}$$

In both (1) and (2), the non-negative integers L_0 and L represent the number of bits in channel 0 and k, respectively, that proceed the detected bits $b_{0,0}$. The received dense WDM equivalent lowpass signal at the input of the FP filter is given by

$$r(t) = \sqrt{P}\, b_0(t) + \sum_{\substack{k=-M/2 \\ k\neq 0}}^{M/2} \sqrt{P}\, b_k(t) \tag{4}$$

where P is the received optical power.

B. FABRY-PEROT FILTERED OUTPUT SIGNAL

The FP filter can be characterized by the following equivalent lowpass transfer function [Ref. 1,7]

$$H(f) = \frac{1-\rho}{1-\rho e^{-j2\pi f/FSR}} \bullet \frac{1-A-\rho}{1-\rho}$$

$$H(f) = \frac{1-\rho}{1-\rho\cos(\frac{2\pi f}{FSR})+j\rho\sin(\frac{2\pi f}{FSR})} \bullet \frac{1-A-\rho}{1-\rho} \tag{5}$$

where ρ is the power reflectivity, A is the power absorption loss (zero for an ideal FP filter) and FSR is the free spectral range. For $|f| < FSR/20\pi$ and assuming $A = 0$, we can approximate $H(f)$ as follow:

$$H(f) \approx \frac{1-\rho}{(1-\rho)+j\frac{2\pi\rho f}{FSR}} = \frac{1}{1+j\frac{2\pi\rho f}{(1-\rho)FSR}}$$

$$\approx \frac{1}{1+j\frac{2\pi f}{c}}, \; |f| < FSR/20\pi \tag{6a}$$

where

$$c = \frac{FSR(1-\rho)}{\rho} \tag{6b}$$

The free spectral range FSR can be related to the full width at half maximum (FWHM) bandwidth B and the finess F of the FP filter as

$$FSR = \frac{\pi\sqrt{\rho}\,B}{1-\rho} = BF \tag{7}$$

Thus if the signal is bandlimited to $|f| < FSR/20\pi$, we can truly approximate (5) with a single-pole RC filter with the following transfer function and impulse response

$$H(f) = \frac{1}{1+j\frac{2\pi f}{c}} \tag{8}$$

$$h(t) = ce^{-ct}\ , \quad t > 0 \tag{9}$$

Figures 2a-b show the magnitude and phase (radians) of $H(f)$ of the FP filter in (5) and its single-pole RC filter approximation given in (8) for $\rho = 0.99$, $F = 312.6$, $B = 12.1$ GHz and $FSR = 3800$ GHz. Note that as the frequency increases, the phases of the FP filter and the RC filter differ markedly, but the magnitudes of their transfer functions remain identical and attenuate rapidly. When $|f| > FSR/20\pi$, the magnitude of $H(f)$ is very small, and therefore, the effect of adjacent channel interference beyond this frequency range is negligible. Figure. 3 shows the normalized impulse response of both FP and single-pole RC filters. In summary, the above approximation is valid for dense WDM analysis when the filter finess F is large or equivalently the FWHM bandwidth B is small since the equivalent lowpass signal must be bandlimited to about $|f| < FSR/20\pi$.

This approximation has been used in [Ref. 5] to study spectral efficiency of optical FDM/ASK systems, which involves the evaluation of the decision variable for worst-case analysis using the eye diagram technique. Since we are interested in the detected bit $b_{0,0}$ in the time interval $(0,T)$, we consider the output filtered signal $s(t)$, $0< t < T$ given by

$$s(t) = s_B(t) + s_{ISI}(t) + s_{ACI}(t), \quad 0 < t < T \tag{10}$$

where

$s_B(t)$: desired signal

$s_{ISI}(t)$: intersymbol interference signal

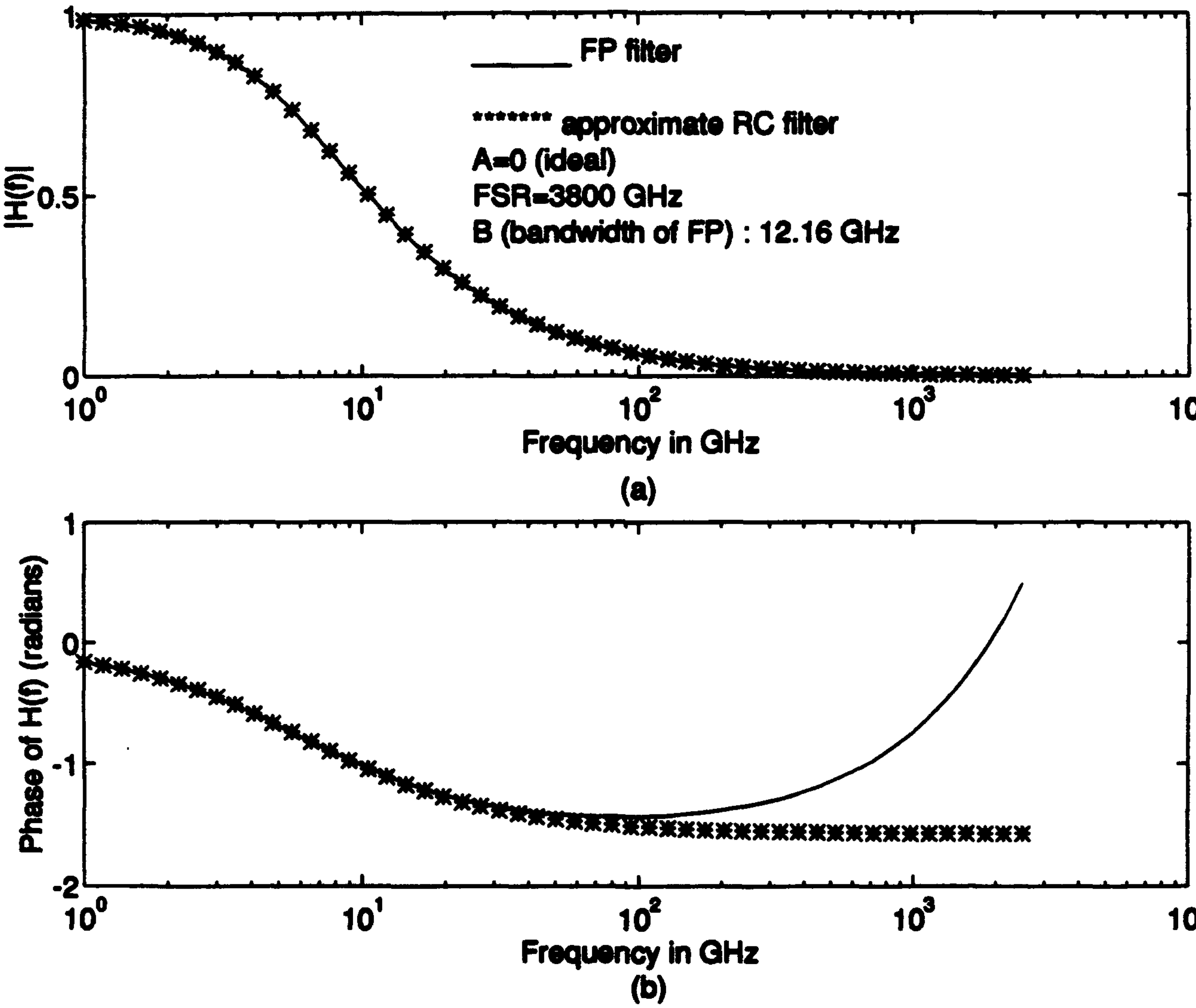


Figure 2: Spectral characteristics of the Fabry-Perot filter and the approximated single-pole RC lowpass filter

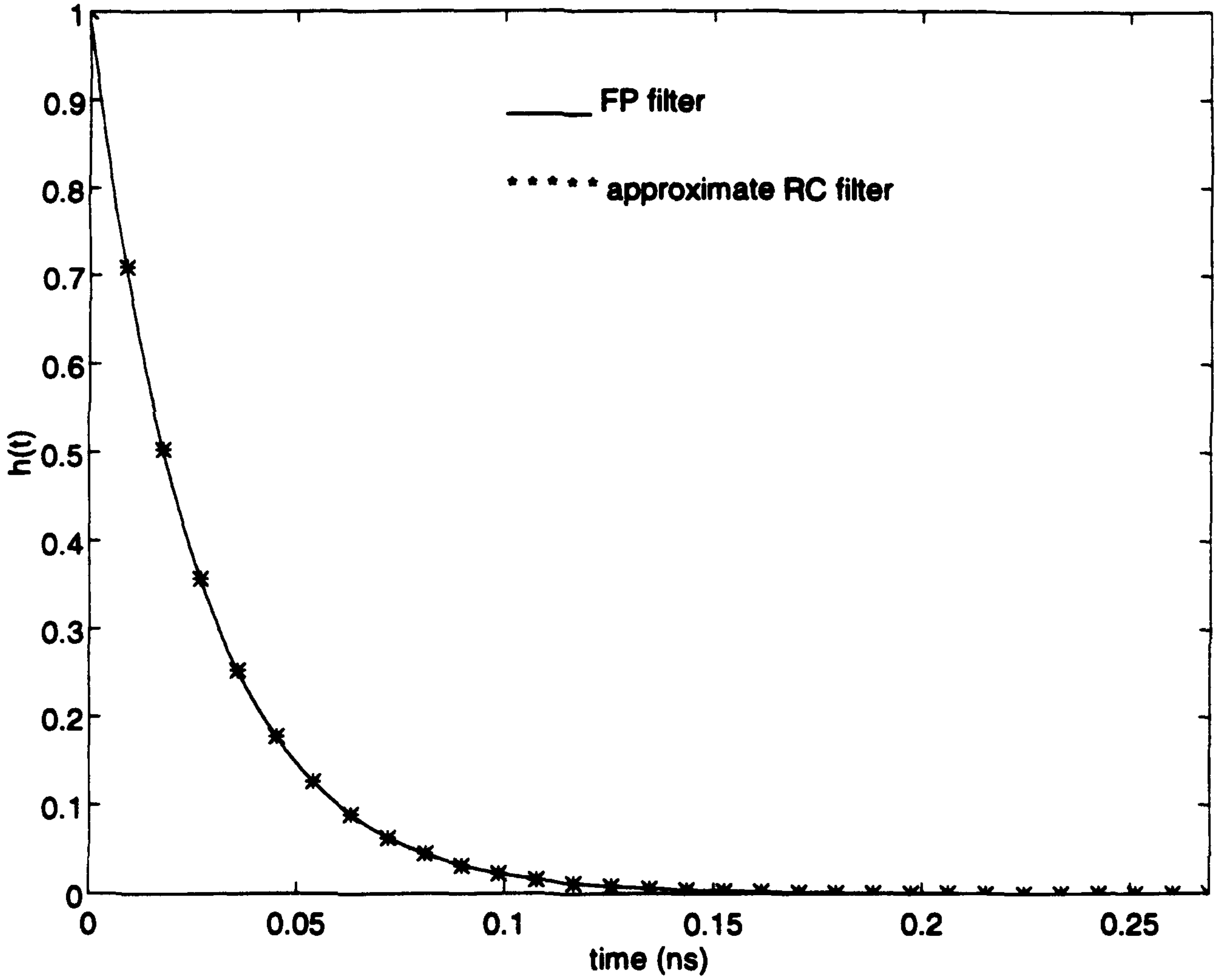


Figure 3: Normalized impulse response of the Fabry-Perot filter and the approximated single-pole RC lowpass filter

$$s_B(t) = \sqrt{P}\, b_{0,0} \int_0^t h(t-\tau)d\tau$$

$$= \sqrt{P}\, b_{0,0}(1-e^{-ct}), \quad 0 < t < T \tag{11}$$

$$s_{ISI}(t) = \sqrt{P} \sum_{i=-L_0}^{-1} b_{0,i} \int_{iT}^{(i+1)T} h(t-\tau)d\tau$$

$$= \sqrt{P}\, e^{-ct} \sum_{i=-L_0}^{-1} b_{0,i}(e^{(i+1)cT} - e^{icT}), \; 0 < t < T \tag{12}$$

$$s_{ACI}(t) = \sqrt{P} \sum_{\substack{k=-M/2 \\ k\neq 0}}^{M/2} \{[\sum_{l=-L}^{-1} b_{k,l} \int_{lT}^{(l+1)T} h(t-\tau)e^{j\omega_k \tau}d\tau]$$

$$+b_{k,0} \int_0^t h(t-\tau)e^{j\omega_k \tau}d\tau\}, \quad 0 < t < T$$

$$= \sqrt{P}\, ce^{-ct} \sum_{\substack{k=-M/2 \\ k\neq 0}}^{M/2} \frac{1}{c+j\omega_k} \{[\sum_{l=-L}^{-1} b_{k,l}(e^{(c+j\omega_k)(l+1)T}$$

$$-e^{(c+j\omega_k)lT})] + b_{k,0}(e^{(c+j\omega_k)t} - 1)\}, \; 0 < t < T \tag{13}$$

The FP filtered output $s(t)$ is detected by the photodetector which produces a current of $\mathcal{R} \mid s(t) \mid^2$ Amps. This current plus additive white postdetection thermal noise current from the amplifier is integrated by the integrator to obtain a decision variable for the threshold detector.

C. DECISION VARIABLES

The decision variable Y appearing at the integrator output consists of the signal component X and noise component N

$$Y = X + N \tag{14}$$

where

$$X = \int_0^T \mathcal{R} |s(t)|^2 dt \tag{15}$$

$$N = \int_0^T n(t)\, dt \tag{16}$$

We note that N is a zero mean Gaussian random variable with variance $N_0 T$. Substituting (10)-(13) into (15) we obtain the signal component X as a function of the two parameters cT and $\omega_k T$, which represent the effect of intersymbol interference and adjacent channel interference, respectively.

$$X = \mathcal{R} PTb_{0,0}^2 [1 - \tfrac{2}{cT}(1 - e^{-cT}) + \tfrac{1}{2cT}(1 - e^{-2cT})]$$

$$+ \mathcal{R} \frac{PT}{2cT}(1 - e^{-2cT}) [\sum_{i=-L_0}^{-1} b_{0,i}(e^{(i+1)cT} - e^{icT})]^2$$

$$+ \mathcal{R} \frac{PT}{2cT}(1 - e^{-2cT}) | \sum_{\substack{k=-M/2 \\ k \neq 0}}^{M/2} \sum_{l=-L}^{-1} \frac{b_{k,l}}{1 + j\omega_k T / cT}$$

$$(e^{(cT + j\omega_k T)(l+1)} - e^{(cT + j\omega_k T) l}) |^2$$

$$+ \Re \frac{PT}{cT} \sum_{\substack{k=-M/2 \\ k\neq 0}}^{M/2} \sum_{\substack{m=-M/2 \\ m\neq 0}}^{M/2} \frac{b_{k,0} b^{*}_{m,0}}{(1+j\omega_k T/cT)(1-j\omega_m T/cT)}$$

$$\{ \left[\begin{array}{ll} cT, & \omega_k = \omega_m \\ \frac{e^{j(\omega_k T - \omega_m T)}-1}{j(\omega_k T - \omega_m T)/cT}, & \omega_k \neq \omega_m \end{array} \right] + \frac{e^{-(cT-j\omega_k T)}-1}{1-j\omega_k T/cT}$$

$$+ \frac{e^{-(cT+j\omega_m T)}-1}{1+j\omega_m T/cT} + \frac{1}{2}(1 - e^{-2cT}) \}$$

$$+ \frac{2PT}{cT} Re\{ \sum_{\substack{k=-M/2 \\ k\neq 0}}^{M/2} \sum_{\substack{m=-M/2 \\ m\neq 0}}^{M/2} \sum_{l=-L}^{-1} \frac{b_{k,l} b^{*}_{m,0}}{(1+j\omega_k T/cT)(1-j\omega_m T/cT)}$$

$$(e^{(cT+j\omega_k T)(l+1)} - e^{(cT+j\omega_k T)l}) \left[\frac{1-e^{-(cT+j\omega_m T)}}{1+j\omega_m T/cT} \right.$$

$$\left. -\frac{1}{2}(1 - e^{-2cT}) \right] \} + \frac{PTb_{0,0}}{cT}(1 + e^{-2cT} - 2e^{-cT})$$

$$\sum_{i=-L_0}^{-1} b_{0,i}(e^{(i+1)cT} - e^{icT}) + \frac{PTb_{0,0}}{cT} Re\{ (1 + e^{-2cT}$$

$$-2e^{-cT}) \sum_{\substack{k=-M/2 \\ k\neq 0}}^{M/2} \sum_{l=-L}^{-1} \frac{b_{k,l}}{1+j\omega_k T/cT} (e^{(cT+j\omega_k T)(l+1)}$$

$$-e^{(cT+j\omega_k T)l}) + 2 \sum_{\substack{k=-M/2 \\ k\neq 0}}^{M/2} \frac{b_{k,0}}{1+j\omega_k T/cT} \left(\frac{e^{j\omega_k T}-1}{j\omega_k T/cT} + e^{-cT} \right.$$

$$-\frac{1}{2}-\frac{1}{2}e^{-2cT}+\frac{e^{-(cT-j\omega_k T)}-1}{1-j\omega_k T/cT})\}+\frac{PT}{cT}\sum_{i=-L_0}^{-1} b_{0,i}(e^{(i+1)cT}$$

$$-e^{icT})Re\{\sum_{\substack{k=-M/2\\k\neq 0}}^{M/2}\ \sum_{l=-L}^{-1}\frac{b_{k,l}}{1+j\omega_k T/cT}(1-e^{-2cT})$$

$$(e^{(cT+j\omega_k T)(l+1)}-e^{(cT+j\omega_k T)l})$$

$$+\sum_{\substack{k=-M/2\\k\neq 0}}^{M/2}\frac{b_{k,0}}{1+j\omega_k T/cT}(\frac{2(1-e^{-(cT-j\omega_k T)})}{1-j\omega_k T/cT}+e^{-2cT}-1)\} \qquad (17)$$

D. BIT ERROR PROBABILITY

For a detection threshold α and an ISI/ACI bit pattern $b = \{b_{k,l}, b_{0,i}\}$; $i = -L_0, \ldots -1$; $l = -L, \ldots, 0$; $k = -M/2, \ldots, M/2$, $k \neq 0$; the conditional bit error probability of the OOK signal represented by the Gaussian random variable Y in (14)-(16) is given by [Ref. 8]

$$P_e(b)=\frac{1}{2}Q(\frac{X_1(b)-\alpha}{\sqrt{N_0T}})+\frac{1}{2}Q(\frac{\alpha-X_0(b)}{\sqrt{N_0T}}) \qquad (18)$$

where Q (X) is defined as

$$Q(x)=\frac{1}{\sqrt{2\pi}}\int_x^{\infty} e^{-y^2/2}dy \qquad (19)$$

and X_0 and X_1 are the values of X in (17) for $b_{0,0} = 0$ and $b_{0,0} = 1$, respectively. The average bit error probability P_e is obtained by taking the expected value of P_e (b) in (18)

over all bit patterns b. The minimum bit error probability is obtained by optimizing over the threshold α. In summary, given α, we calculate the following expectations:

$$P_e = \underset{b}{E}\ \{P_e(b)\} \tag{20}$$

$$P_{e,\min} = \min_{\alpha} \underset{b}{E}\ \{P_e(b)\} = \min_{\alpha} \frac{1}{2^{M(L+1)+L_0}} \sum_{2^M\, patterns} p(b)$$

where

$$p(b) = \frac{1}{2}\left(\frac{1}{2\pi}\right)^M \Big\{ \underbrace{\int_0^{2\pi} \cdots \int_0^{2\pi}}_{M} Q\left(\frac{\alpha - X_0(\Phi_{-M/2} \ldots \Phi_{M/2})}{\sqrt{N_0 T}}\right) d\Phi_{-M/2} \ldots d\Phi_{M/2}$$

$$+ \underbrace{\int_0^{2\pi} \cdots \int_0^{2\pi}}_{M} Q\left(\frac{X_1(\Phi_{-M/2} \ldots \Phi_{M/2}) - \alpha}{\sqrt{N_0 T}}\right) d\Phi_{-M/2} \ldots d\Phi_{M/2} \Big\} \tag{21}$$

III. NUMERICAL RESULTS

In this section we present the numerical results for a) bit error probability versus signal-to-noise ratio $Z = \mathcal{R}P\sqrt{T/N_0}$ as a function of the normalized channel spacing (normalized to the bit rate)

$$I = \frac{\omega_k T}{2\pi k} = (\Delta f)\,T \tag{22}$$

where $\Delta f = \omega_k / 2\pi k$ is the equal channel spacing in GHz, and b) power penalty versus normalized channel spacing I as a function of the FP filter parameter $cT = (\pi/\sqrt{\rho})BT$ where B is the filter full width at half maximum bandwidth(FWHM).

Figures 4-5 show the minimum bit error probability versus signal-to-noise ratio (Z) for $cT = 5$ and 10, respectively, and that of a single channel (SC) operation without filtering or with filtering but without *ISI*. In Fig. 4 we observe that a large degradation occurs due to *ISI* for $cT = 5$ which represents a narrowband filter. As the FP filter bandwidth is made larger as in Fig. 5 with $cT = 10$, the *ISI* is reduced but the *ACI* increases.

In our model, we are constrained to $M = 4$ for the case under consideration. We set a FP filter with free spectral range, FWHM B=121.6 GHz, FSR = 3800 GHz, finess $F = FSR/B$ = 312.6, and c = 38.4 GHz, $1/T$ = 2.56 Gb/s, then cT = 15.

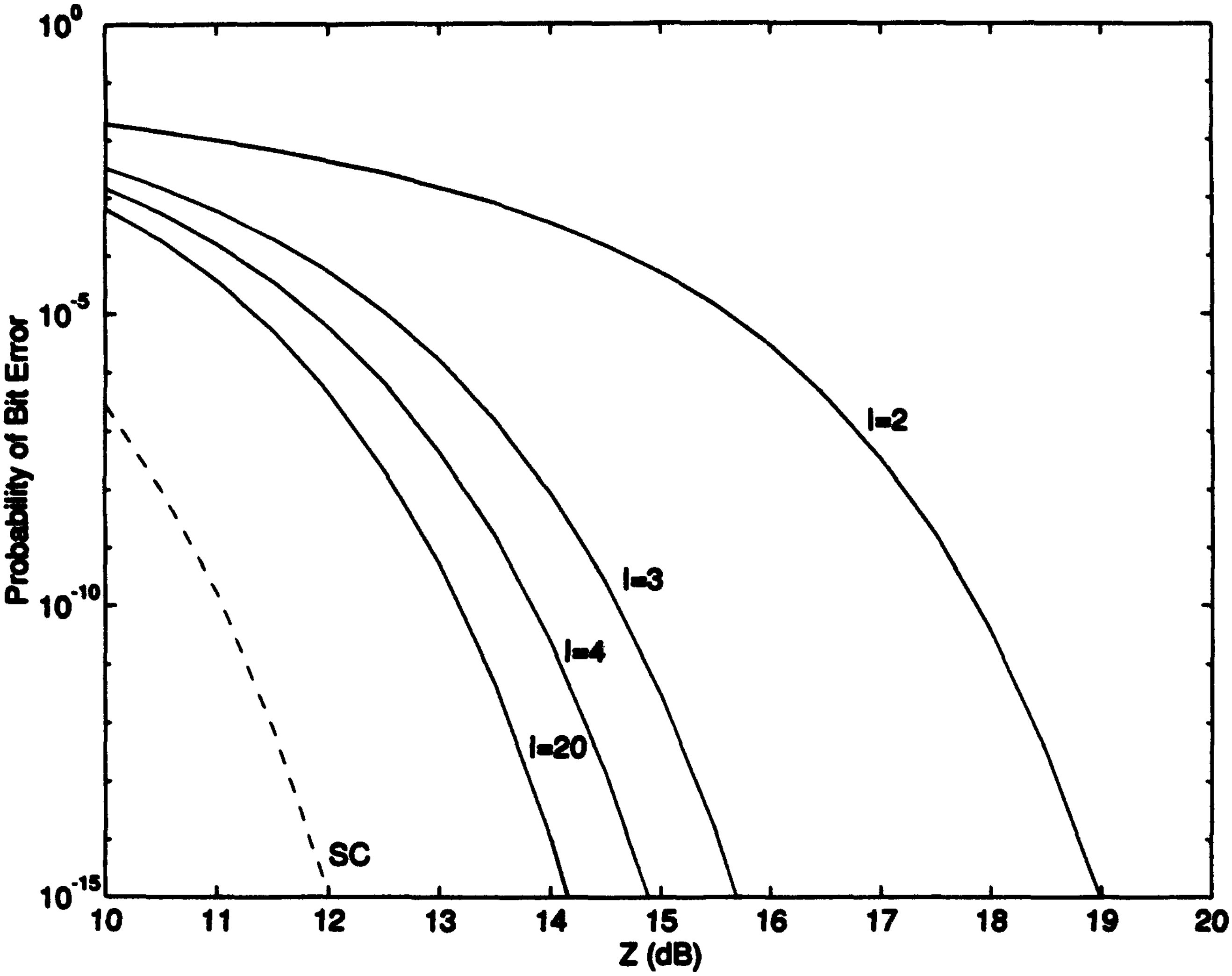


Figure 4: Probability of bit error versus signal-to-noise ratio as a function of normalized channel spacing for cT=5

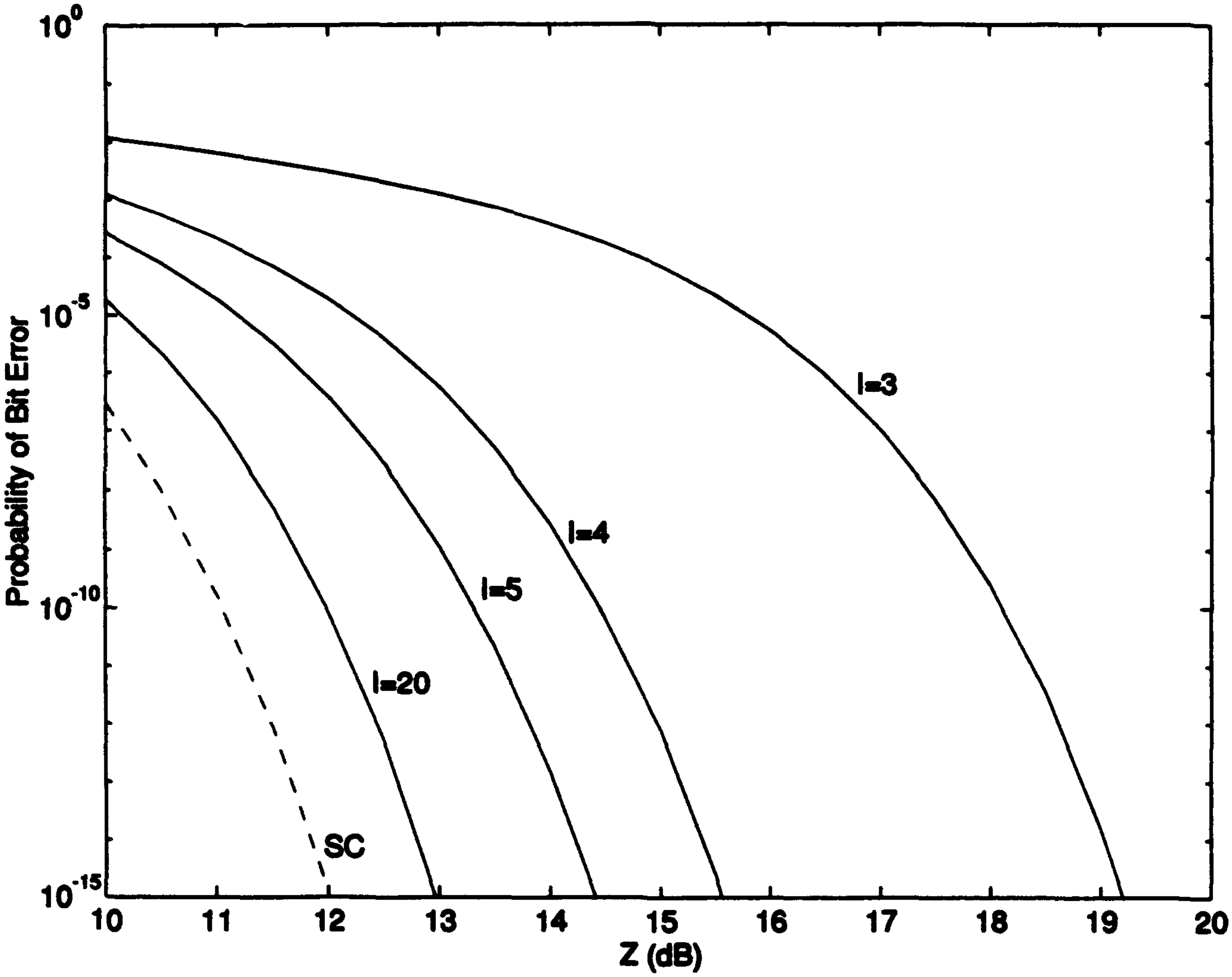


Figure 5: Probability of bit error versus signal-to-noise ratio as a function of normalized channel spacing for cT=10

(i.e. channel spacing is 12 times the bit rate or 30.4 GHz). In this model the farthest adajcent channel for $M = 4$ is twice the channel spacing which is 60.8 GHz. This verifies the assumption $|f-f_0| < FSR/20\pi = 60.5$ GHz, where f_0 is the FP filter center frequency. This result agrees well with that in [Ref. 6; Figs. 6,9, $M/F = 0.4$, a = 0.2]. Thus we incorporate the degradation caused by the four nearest adjacent channels. We observe that for bit rates of 1 Gb/s or less, our model is valid up to $M = 10$, and very little difference is observed between $M = 4$ and $M = 10$. Also, we observe that there is little difference between $M = 2$ and $M = 4$ when $I >= 10$ for bit rates up to 3 Gb/s. In all results we set $L_0 = 2$ and $L = 0$.

Figure 6 also shows the power penalty for a dense WDM system relative to a single channel operation at the minimum bit error probability of 10^{-15}. This is the required additional signal power (dBW) for the dense WDM system to be able to operate at the 10^{-15} bit error probability achieved in the single channel system with a SNR=12dB. The dense WDM system is *ISI*-limited at 2.2 dB, 1 dB, 0.5 dB, and 0.4 dB in power penalty for $cT = 5$, 10, 15, and 20, respectively. It is seen that for a 2.3 dB power penalty, the normalized channel spacing can be as close as $I = 6$ (i.e., a channel spacing of six times the bit rate) for $cT = 5$. If the power penalty criterion is 1 dB, the normalized channel spacing is I=12 for $cT = 10$, 15, 20. We remark that although the exact transfer functions of the FP filter is used in [Ref. 6], a number of approximation have been made to obtain numerical results. The approximations are 1) the *ISI* is obtained by modeling FP filter as a single-pole RC filter [Ref. 6, Eqs. (4) and (36)], 2) approximating the finite integration

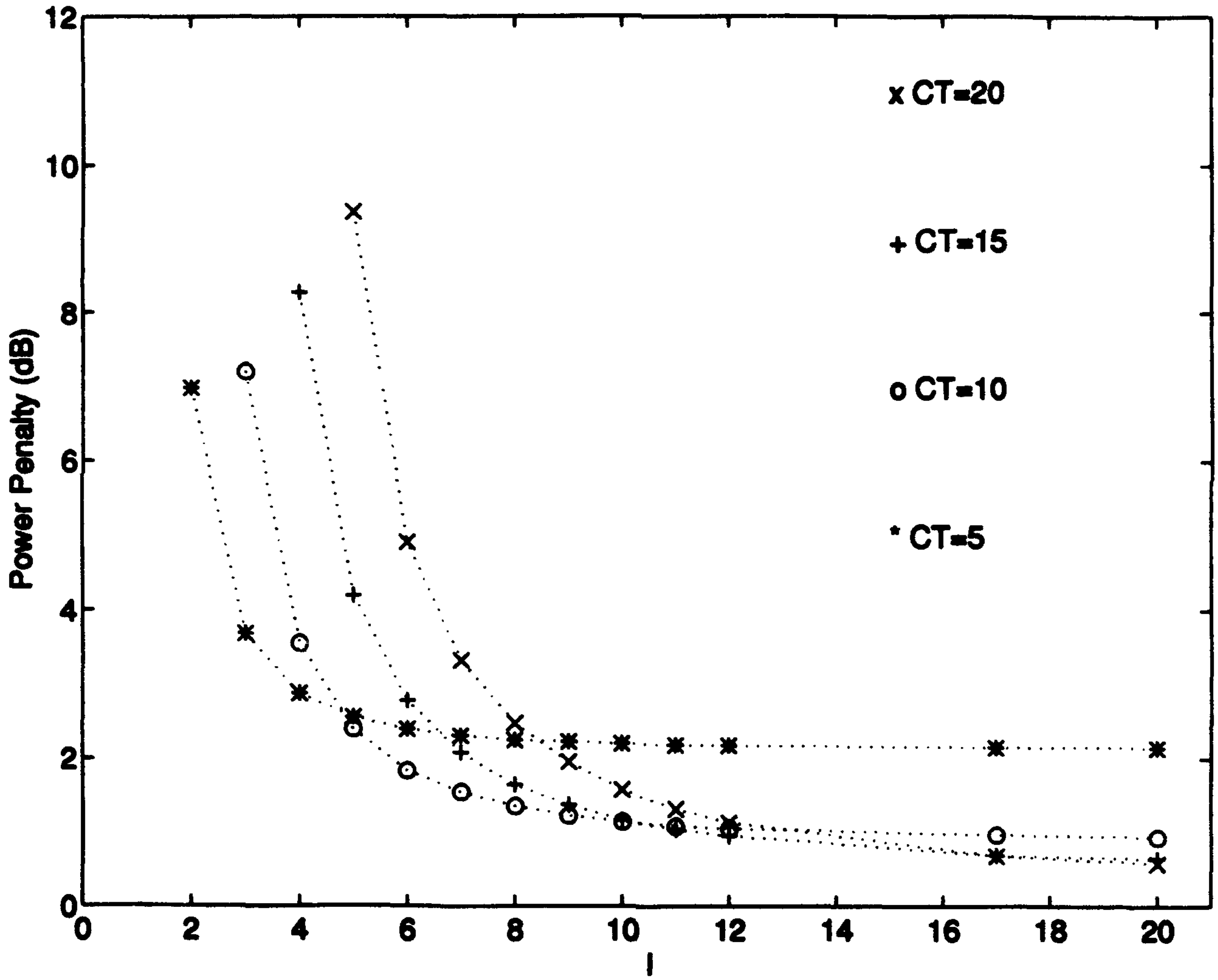


Figure 6: Power penalty versus normalized channel spacing as a function of Fabry-Perot filter parameter cT

with an infinite integration in the calculation of *ACI* [Ref. 6, Eq. (15)], and 3) the beat interference is ignored. On the other hand, the *ISI* and *ACI* in our investigation are obtained by modeling the FP filter as a single-pole RC filter, using finite integration and including the beat interference. Since the results in our investigation and in [Ref. 6] agree well, we conclude that approximations are quite valid. We also note that our results also agree well with the simulation carried out in [Ref. 1, Fig. 17].

The above numerical results shown in Figs. 4-6 are obtained with an optimized threshold setting. Figure 7 shows the power penalty for fixed threshold $\alpha = \mathcal{R}PT/2$ which is the same optimum threshold for single channel operation (midpoint between the received power for bit zero and bit one). It is seen that the performance of a dense WDM system is quite sensitive to α for a narrow band filter. An additional 1.8 dB is observed for $cT = 5$ for $I > 8$, and 0.5 dB for $cT = 10$ for $I > 12$. Negligible degradation is observed for $cT = 15, 20$ for $I > 16$.

Figures 8-9 show the power penalty versus normalized channel spacing as a function of FP filter parameter cT for the worst-case analysis with optimal threshold and fixed threshold, respectively. The worst-case bit pattern is fixed to produce the minimum X_1 and maximum X_0 where X_1 and X_0 are the values to X in Appendix-A equation (9) with $b_{0,0} = 1$ and $b_{0,0} = 0$ respectively.

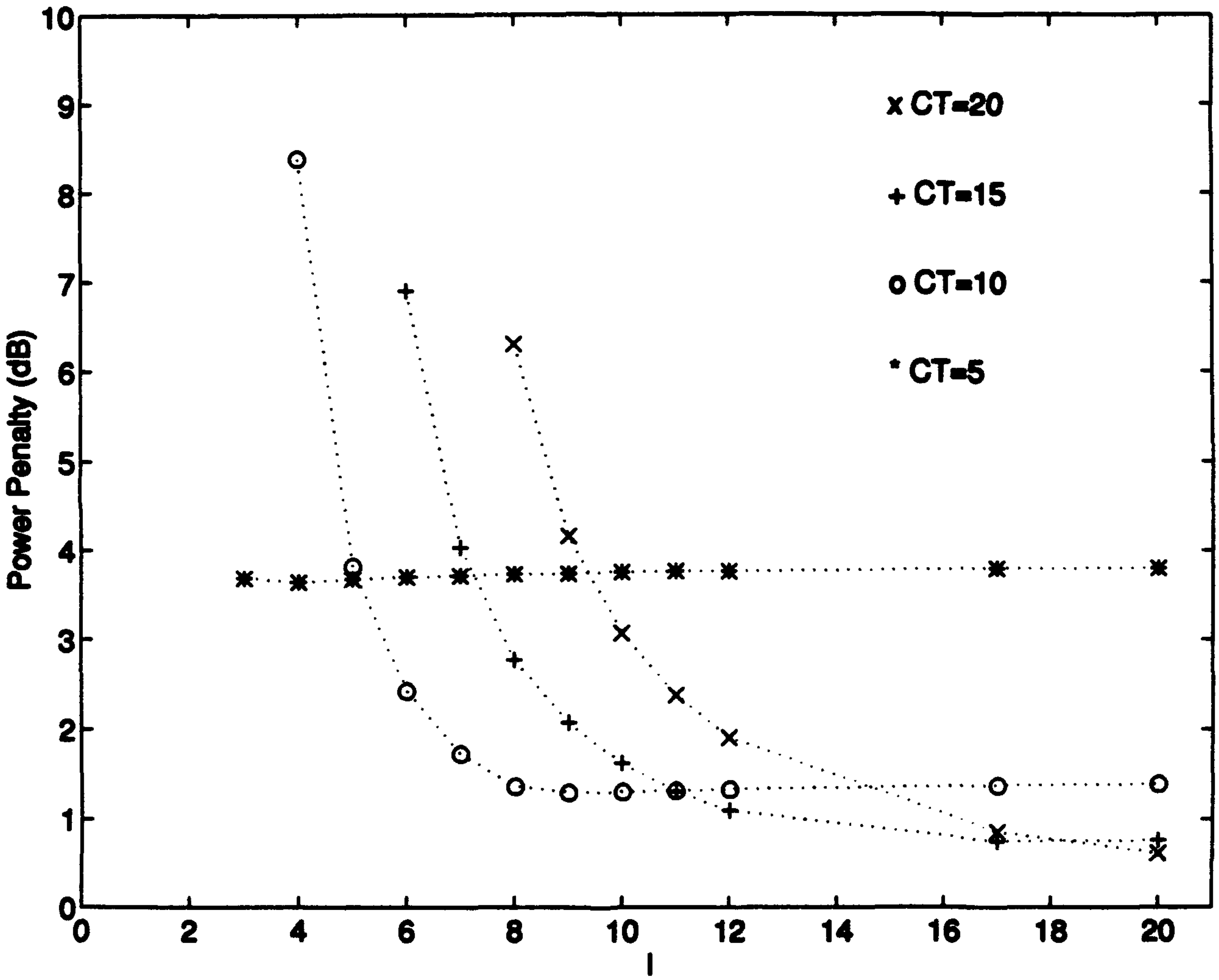


Figure 7: Power penalty versus normalized channel spacing as a function of Fabry-Perot filter parameter cT with a fixed threshold α=0.5

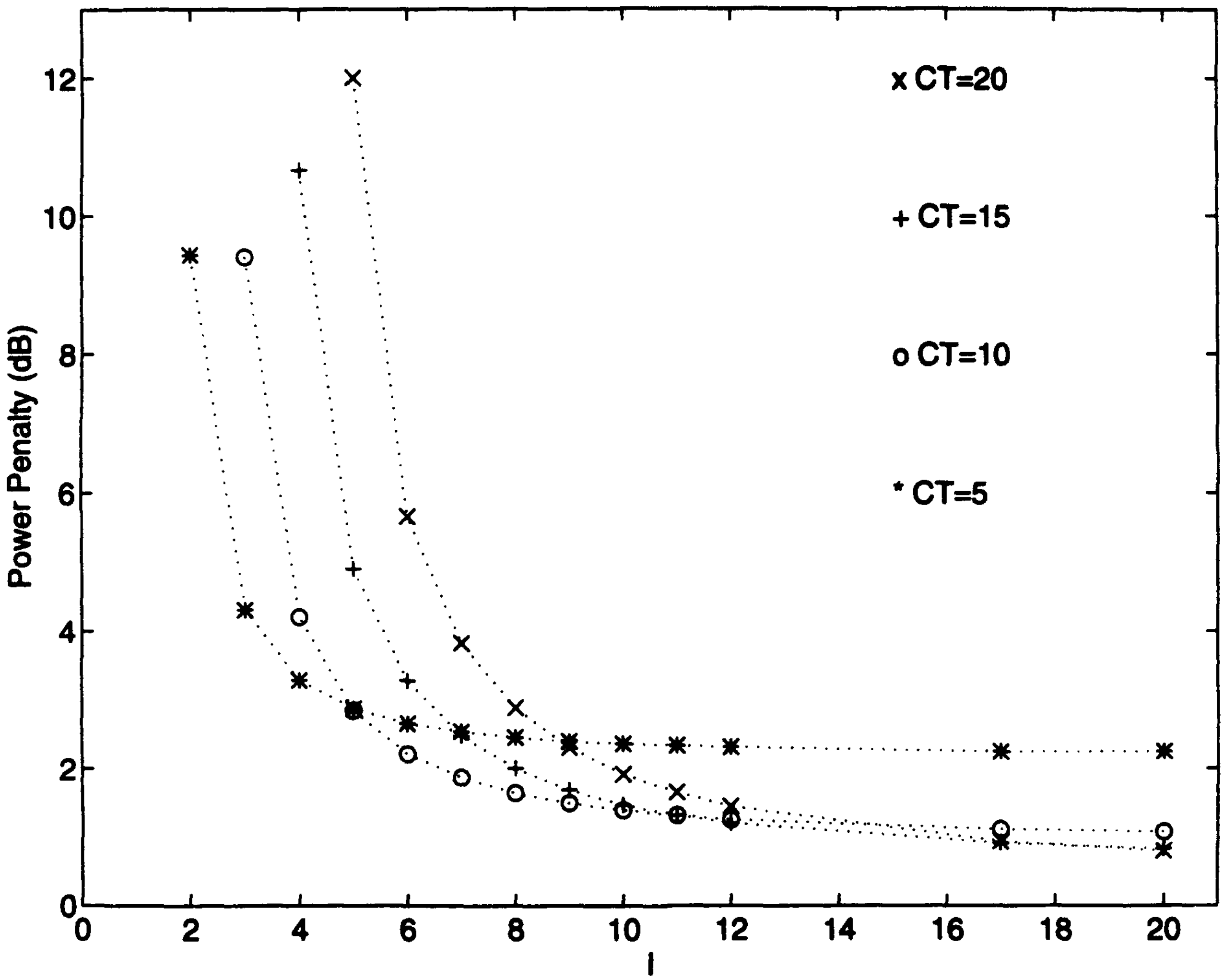


Figure 8: Worst-case power penalty versus normalized channel spacing as a function of Fabry-Perot filter parameter cT with optimal threshold $\alpha=(X_0+X_1)/2$

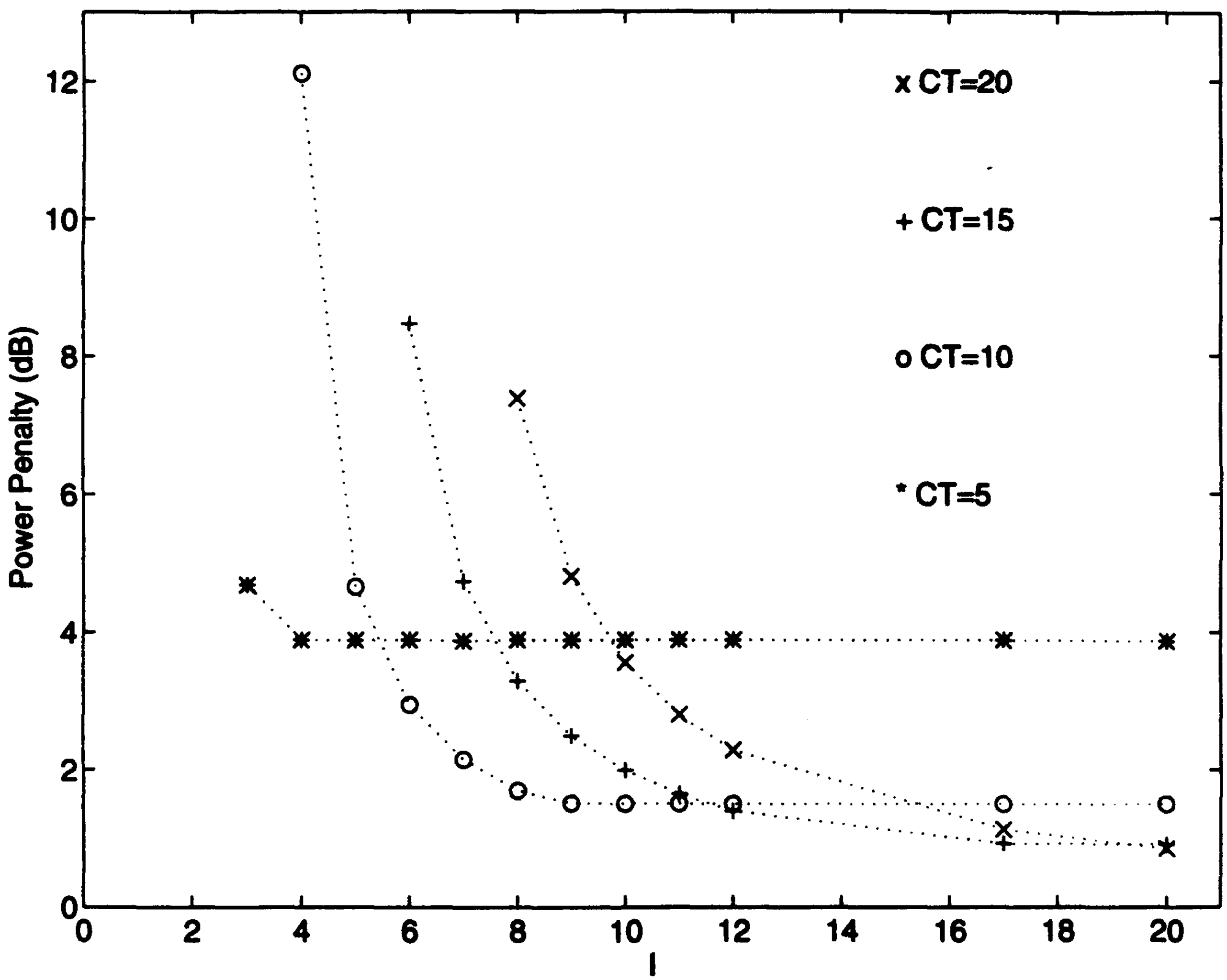


Figure 9: Worst-case power penalty versus normalized channel spacing as a function of Fabry-Perot filter parameter cT with fixed threshold α=0.5

We observe that the power penalty for the worst-case analysis is only slightly larger than that of the exact analysis for $I > 10$ shown in Fig. 6. Similarly the power penalty for the worst-case analysis with fixed threshold is only slightly larger than that of the exact analysis with fixed threshold for $I > 10$ shown is Fig. 7. The reason for this is that for large channel spacing ($I > 10$), the *ACI* effect is small, so the *ACI* bit pattern has a small influence on the power penalty.

Figure 10 shows the normalized optimal threshold for the exact analysis shown in Fig. 6. It is observed that $\alpha \approx 0.4$ for $I > 10$. Note that the normalized optimal threshold for the single channel operation is $\alpha = 0.5$.

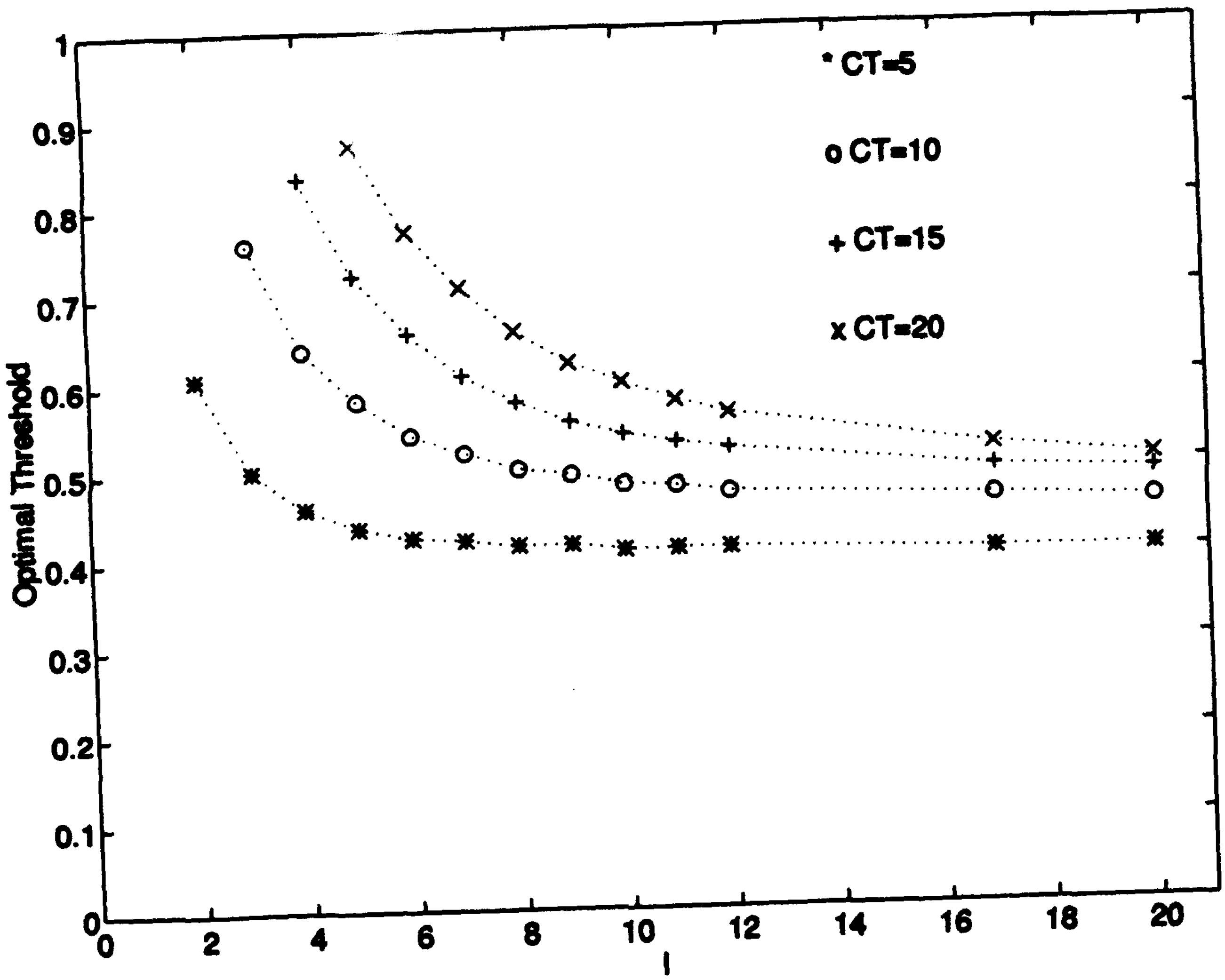


Figure 10: The normalized optimal threshold versus normalized channel spacing as a function of Fabry-Perot filter parameter cT

IV. CONCLUSIONS

We have presented a simple model for the analysis of dense WDM systems employing an external OOK modulator. The only approximation that we use involves the modeling of the Fabry-Perot filter by a single-pole RC filter assuming the equivalent lowpass signal is bandlimited to the frequency range $|f| < FSR / 20\pi$. This model enables us to obtain a closed form expression for the bit error probability which previously can only be obtained via numerical analysis [Ref. 6]. For FP filter with an *FSR* around 3800 GHz, our model can include the *ACI* effects of four adjacent channels for bit rates up to 2.5 Gb/s. Our numerical results show that this model agrees well with that in [Ref. 6].

APPENDIX A

DERIVATION OF FORMULA FOR DECISION VARIABLES

DESIRED CHANNEL : CHANNEL 0

ADJACENT CHANNEL : $k, k = -M/2, \ldots, -1, 1, \ldots, M/2; M: even$

BIT IN CHANNEL 0 IN *ith* TIME INTERVAL $(iT, (i+1)T)$:

$b_{0,i} \in \{0, 1\}, \quad b_{0,0}$: DETECTED BIT IN $(0, T)$

BIT IN CHANNEL k IN *lth* TIME INTERVAL $(lT, (l+1)T)$:

$b_{k,l} \in \{0, e^{j\phi_k}\}$ WHERE $j = \sqrt{-1}$: IMAGINARY NUMBLER

ω_k : FREQUENCY SPACING BETWEEN CHANNEL k AND CHANNEL 0,

$\omega_k = -\omega_{-k}$

ϕ_k : PHASE OFFSET BETWEEN CHANNELS k AND CHANNEL 0,

UNIFORMLY DISTRIBUTED BETWEEN (0, 2π)

DATA SIGNAL IN CHANNEL 0 :

$$b_0(t) = \sum_{i=-L_0}^{0} b_{0,i}\, p_T(t - iT)$$

DATA SIGNAL IN CHANNEL k :

$$b_k(t) = \sum_{l=-L}^{0} b_{k,l} e^{j\omega_k t} p_T(t - lT)$$

L_0 , L : INTEGERS

$$p_T(t) = \begin{cases} 1, & 0<t<T \\ 0, & otherwise \end{cases}$$

$$p_T(t - iT) = \begin{cases} 1, & iT<t<(i+1)T \\ 0, & otherwise \end{cases}$$

THE RECEIVED SIGNAL AT THE INPUT OF THE FP FILTER OF

CHANNEL 0 IS :

$$r(t) = \sqrt{P}\, b_0(t) + \sum_{\substack{k=-M/2 \\ k\neq 0}}^{M/2} \sqrt{P}\, b_k(t)$$

P : RECEIVED OPTICAL POWER.

THE OUTPUT OF THE FP FILTER IS

$$r_0(t) = \int_{-\infty}^{\infty} h(t-\tau) r(\tau) d\tau$$

WHERE $h(t)$ IS THE EQUIVALENT LOWPASS IMPULSE RESPONSE OF

THE FP FILTER OF CHANNEL 0

$$r_0(t) = \sqrt{P} \int_{-\infty}^{\infty} h(t-\tau) b_0(\tau) d\tau + \sqrt{P} \sum_{\substack{k=-M/2 \\ k\neq 0}}^{M/2} \int_{-\infty}^{\infty} h(t-\tau) b_k(\tau) d\tau$$

$$= \sqrt{P}\, b_{0,0} \int_{-\infty}^{\infty} h(t-\tau) p_T(\tau) d\tau$$

$$+ \sqrt{P} \sum_{i=-L_0}^{-1} b_{0,i} \int_{-\infty}^{\infty} h(t-\tau) p_T(\tau - iT) d\tau$$

$$+\sqrt{P}\sum_{\substack{k=-M/2\\k\neq 0}}^{M/2}\sum_{l=-L}^{0} b_{k,l}\int_{-\infty}^{\infty} h(t-\tau)e^{j\omega_k\tau}p_T(\tau-lT)d\tau$$

SINCE THE DETECTION INTERVAL IS $0 < t < T$, WE ONLY NEED TO EVALUATE $s(t) = r_0(t)$, $0 < t < T$

$$s(t) = \sqrt{P}\, b_{0,0}\int_0^t h(t-\tau)d\tau + \sqrt{P}\sum_{i=-L_0}^{-1} b_{0,i}\int_{iT}^{(i+1)T} h(t-\tau)d\tau$$

$$+\sqrt{P}\sum_{\substack{k=-M/2\\k\neq 0}}^{M/2}\{[\sum_{l=-L}^{-1} b_{k,l}\int_{lT}^{(l+1)T} h(t-\tau)e^{j\omega_k\tau}d\tau]$$

$$+b_{k,0}\int_0^t h(t-\tau)e^{j\omega_k\tau}d\tau\}$$

$$0 < t < T$$

$$= s_B(t)+s_{ISI}(t)+s_{ACI}(t) \qquad (1)$$

$s_B(t)$: DESIRED SIGNAL

$s_{ISI}(t)$: INTERSYMBOL INTERFERENCE

$s_{ACI}(t)$: ADJACENT CHANNEL INTERFERENCE

FP FILTER :

LOWPASS EQUIVALENT TRANSFER FUNCTION

$$H(f)=\frac{1-\rho}{1-\rho e^{-j2\pi f/FSR}}\bullet\frac{1-A-\rho}{1-\rho}=\frac{1-\rho}{1-\rho\cos(\frac{2\pi f}{FSR})+j\rho\sin(\frac{2\pi f}{FSR})}\bullet\frac{1-A-\rho}{1-\rho}$$

ρ : POWER REFLECTIVITY

A : POWER ABSORPTION LOSS (A= 0 FOR IDEAL FILTER)

FSR : FREE SPECTRAL RANGE

SINCE $f << FSR$ (FOR OPERATING FREQUENCY RANGE)

WE CAN APPROXIMATE $H(f)$ AS (ASSUME A= 0) :

$$H(f)=\frac{1-\rho}{(1-\rho)+j\frac{2\pi f\rho}{FSR}}=\frac{1}{1+j\frac{2\pi f\rho}{(1-\rho)FSR}}$$

$$H(f)\approx\frac{1}{1+j\frac{2\pi f}{c}}, \qquad \text{WHERE} \qquad c=\frac{FSR(1-\rho)}{\rho}$$

FOR FP FILTER WE ALSO HAVE

$$\frac{FSR}{B}=\frac{\pi\sqrt{\rho}}{1-\rho}$$

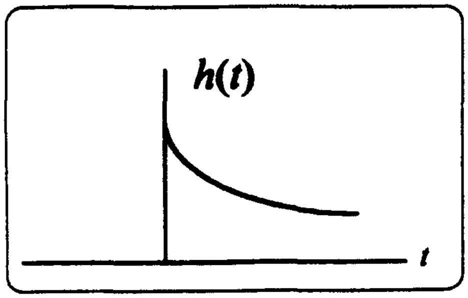
B : FULL WIDTH AT HALF MAXIMUM BANDWIDTH (FWHM) OR HALF POWER BANDWIDTH

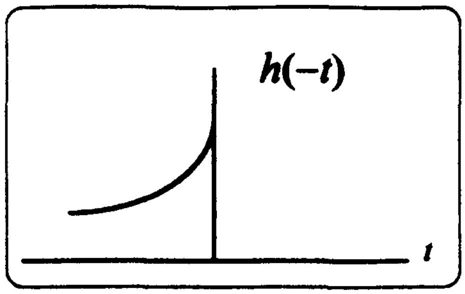
THUS

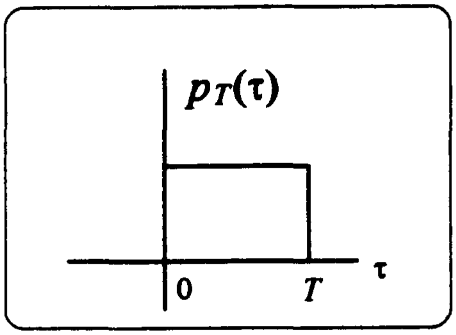
$$h(t)=\begin{cases}ce^{-ct}, & t>0\\ 0, & otherwise\end{cases} \tag{2}$$

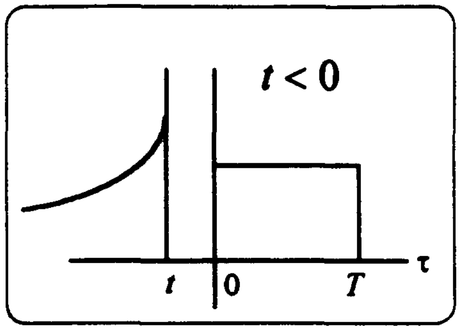
DERIVATION OF EQ.1 ON P. 19

1. $S_B(t)$:

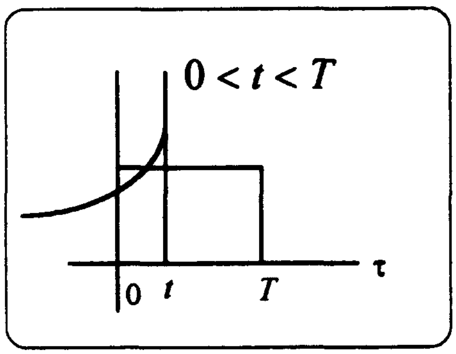


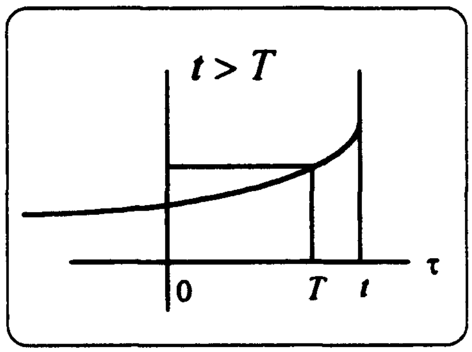






$$\int_{-\infty}^{\infty} h(t-\tau)p_T(\tau) = 0$$





$$\int_{-\infty}^{\infty} h(t-\tau)p_T(\tau)d\tau$$

$$=\int_{0}^{t} h(t-\tau)d\tau$$

$$\int_{-\infty}^{\infty} h(t-\tau)p_T(\tau)d\tau$$

$$=\int_{0}^{T} h(t-\tau)d\tau$$

$$s_B(t) = \sqrt{P}\, b_{0,0} \int_0^t h(t-\tau)d\tau \qquad 0 < t < T \qquad (3)$$

SUBSTITUTING (2) INTO (3) WE OBTAIN

$$s_B(t) = \sqrt{P}\, b_{0,0} \int_0^t ce^{-c(t-\tau)}d\tau = \sqrt{P}\, b_{0,0}\, ce^{-ct} \int_0^t e^{c\tau}d\tau$$

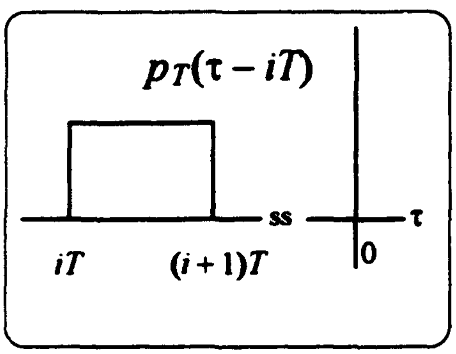
$$= \sqrt{P}\, b_{0,0} ce^{-ct} \tfrac{1}{c} e^{c\tau}\big|_0^t = \sqrt{P}\, b_{0,0} e^{-ct}(e^{ct}-1)$$

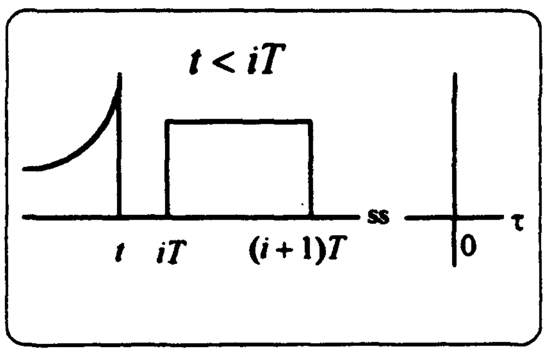
$$= \sqrt{P}\, b_{0,0}(1-e^{-ct}) \qquad 0 < t < T$$

a) BIT 1 : $s_{B,1}(t) = \sqrt{P}(1-e^{-ct}) \qquad 0 < t < T$

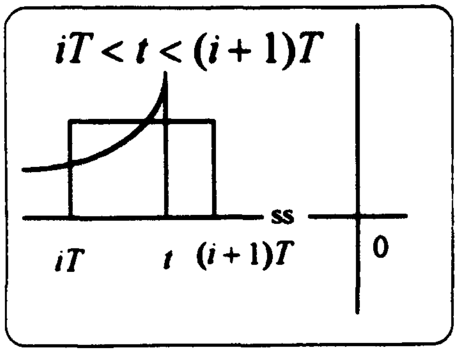
b) BIT 0 : $s_{B,0}(t) = 0 \qquad 0 < t < T$

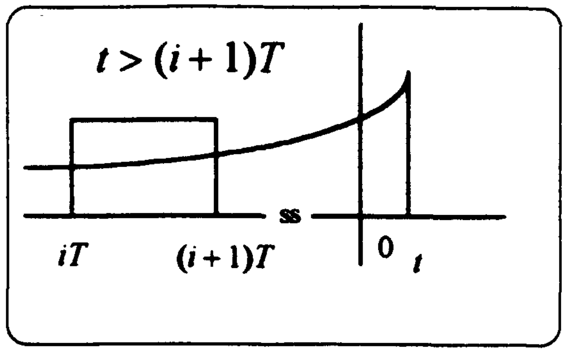
2. $S_{ISI}(t)$:





$$\int_{-\infty}^{\infty} h(t-\tau)p_T(\tau - iT)d\tau = 0$$





$$\int_{-\infty}^{\infty} h(t-\tau)\, p_T(\tau - iT)\, d\tau \qquad\qquad \int_{-\infty}^{\infty} h(t-\tau)\, p_T(\tau - iT)\, d\tau$$

$$= \int_{iT}^{t} h(t-\tau)\, d\tau \qquad\qquad = \int_{iT}^{(i+1)T} h(t-\tau)\, d\tau$$

$$s_{ISI}(t) = \sqrt{P} \sum_{i=-L_0}^{-1} b_{0,i} \int_{iT}^{(i+1)T} h(t-\tau)\, d\tau \quad 0 < t < T \tag{5}$$

SUBSTITUTING (2) INTO (5) WE OBTAIN

$$s_{ISI}(t) = \sqrt{P} \sum_{i=-L_0}^{-1} b_{0,i} \int_{iT}^{(i+1)T} c e^{-c(t-\tau)}\, d\tau$$

$$= \sqrt{P} \sum_{i=-L_0}^{-1} b_{0,i}\, c e^{-ct} \frac{e^{c\tau}}{c} \Big|_{iT}^{(i+1)T}$$

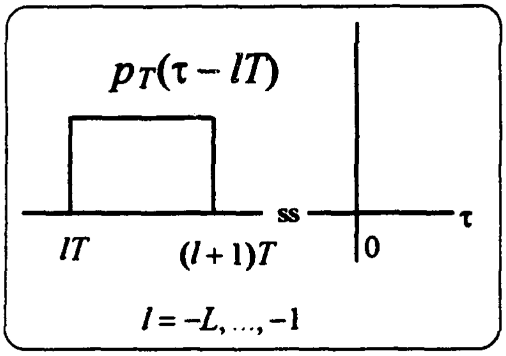
$$s_{ISI}(t) = \sqrt{P} \sum_{i=-L_0}^{-1} b_{0,i}\, e^{-ct} \left(e^{(i+1)cT} - e^{icT}\right)$$

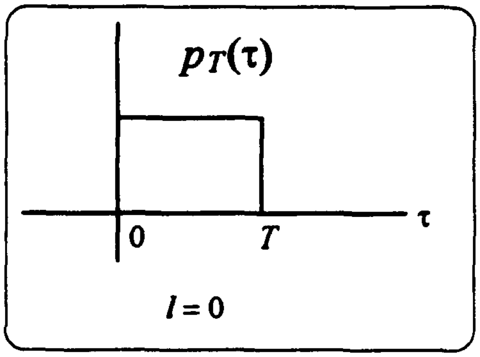
$$s_{ISI}(t) = \sqrt{P}\, e^{-ct} \sum_{i=-L_0}^{-1} b_{0,i}(e^{(i+1)cT} - e^{icT}) \quad 0 < t < T \qquad (6)$$

<u>WORST-CASE ISI</u> : $b_{0,i} = b_- \quad , \quad L_0 = \infty$

$$s_{ISI}^{wc}(t) = \sqrt{P}\, b_- e^{-ct} \qquad 0 < t < T \qquad \textbf{(6a)}$$

3. $s_{ACI}(t)$:





SO THIS IS THE COMBINATION OF THE ABOVE 2 CASES

$$s_{ACI}(t) = \sqrt{P} \sum_{\substack{k=-M/2 \\ k\neq 0}}^{M/2} \{[\sum_{l=-L}^{-1} b_{k,l} \int_{lT}^{(l+1)T} h(t-\tau)e^{j\omega_k\tau}d\tau]$$

$$+b_{k,0} \int_0^t h(t-\tau)e^{j\omega_k\tau}d\tau\} \qquad 0 < t < T \qquad (7)$$

SUBSTITUTING (2) INTO (7) WE OBTAIN

$$s_{ACI}(t) = \sqrt{P} \sum_{\substack{k=-M/2 \\ k\neq 0}}^{M/2} \{[\sum_{l=-L}^{-1} b_{k,l} \int_{lT}^{(l+1)T} ce^{-c(t-\tau)}e^{j\omega_k\tau}d\tau]$$

$$+b_{k,0} \int_{0}^{t} ce^{-c(t-\tau)}e^{j\omega_k\tau}d\tau\}$$

$$= \sqrt{P} \sum_{\substack{k=-M/2 \\ k\neq 0}}^{M/2} \{[\sum_{l=-L}^{-1} b_{k,l}\, ce^{-ct} \int_{lT}^{(l+1)T} e^{(c+j\omega_k)\tau}d\tau$$

$$+b_{k,0}\, ce^{-ct} \int_{0}^{t} e^{(c+j\omega_k)\tau}d\tau\}$$

$$= \sqrt{P} \sum_{\substack{k=-M/2 \\ k\neq 0}}^{M/2} \{[\sum_{l=-L}^{-1} b_{k,l}ce^{-ct}\frac{e^{(c+j\omega_k)\tau}}{c+j\omega_k}\Big|_{lT}^{(l+1)T}]$$

$$+b_{k,0}ce^{-ct}\frac{e^{(c+j\omega_k)\tau}}{c+j\omega_k}\Big|_{0}^{t}\}$$

$$s_{ACI}(t) = \sqrt{P}\, ce^{-ct} \sum_{\substack{k=-M/2 \\ k\neq 0}}^{M/2} \frac{1}{c+j\omega_k}\{[\sum_{l=-L}^{-1} b_{k,l}(e^{(c+j\omega_k)(l+1)T} - e^{(c+j\omega_k)lT})]$$

$$+b_{k,0}(e^{(c+j\omega_k)t} - 1)\} \qquad 0 < t < T \qquad (8)$$

<u>WORSE-CASE *ACI*</u> : $b_{k,l} = b_{k,0} = b, L = \infty$

$$s^{wc}_{ACI}(t) = \sqrt{P}\, ce^{-ct} \sum_{\substack{k=-M/2 \\ k\neq 0}}^{M/2} \frac{be^{(c+j\omega_k)t}}{c(1+j\frac{\omega_k}{c})}$$

$$s^{wc}_{ACI}(t) = \sqrt{P}\, b \sum_{\substack{k=-M/2 \\ k\neq 0}}^{M/2} \frac{e^{j\omega_k t}}{1+j\frac{\omega_k}{c}} \qquad 0 < t < T \tag{8a}$$

OUTPUT OF PHOTODETECTOR : $\mathcal{R}\ |s(t)|^2$

SIGNAL OUTPUT OF INTEGRATOR : $X = \mathcal{R}\int_0^T |s(t)|^2 dt$

$$|s(t)|^2 = |s_B(t) + s_{ISI}(t) + s_{ACI}(t)|^2$$

$$= |s_B(t)|^2 + |s_{ISI}(t)|^2 + |s_{ACI}(t)|^2 + 2Re\{s_B(t)s^*_{ISI}(t)\}$$

$$+2Re\{s^*_B(t)s_{ACI}(t)\} + 2Re\{s^*_{ISI}(t)s_{ACI}(t)\}$$

WORSE-CASE

1) $s_B(t) = \sqrt{P}\, b_{0,0}(1 - e^{-ct}) \qquad 0 < t < T$

$$|s_B(t)|^2 = Pb^2_{0,0}(1 - e^{-ct})^2 = Pb^2_{0,0}(1 - 2e^{-ct} + e^{-2ct}) \qquad 0 < t < T$$

$$\int_0^T |s_B(t)|^2 dt = PTb^2_{0,0} - 2Pb^2_{0,0}\int_0^T e^{-ct}dt + Pb^2_{0,0}\int_0^T e^{-2ct}dt$$

$$= PTb_{0,0}^2 - 2Pb_{0,0}^2 \frac{1}{-c} e^{-ct} |_0^T + Pb_{0,0}^2 \frac{1}{-2c} e^{-2ct} |_0^T$$

$$= PTb_{0,0}^2 + \frac{2Pb_{0,0}^2}{c}(e^{-cT} - 1) - \frac{Pb_{0,0}^2}{2c}(e^{-2cT} - 1)$$

$$= PTb_{0,0}^2[1 + \frac{2}{cT}(e^{-cT} - 1) - \frac{1}{2cT}(e^{-2cT} - 1)]$$

$$= PTb_{0,0}^2[1 - \frac{2}{cT}(1 - e^{-cT}) + \frac{1}{2cT}(1 - e^{-2cT})]$$

2) $s_{ISI}^{wc}(t) = \sqrt{P}\, b_- e^{-ct} \qquad 0 < t < T$

$$|s_{ISI}^{wc}(t)|^2 = Pb_-^2 e^{-2ct} \qquad 0 < t < T$$

$$\int_0^T |s_{ISI}^{wc}(t)|^2 \, t = Pb_-^2 \int_0^T e^{-2ct} dt = Pb_-^2 \frac{1}{-2c} e^{-2ct} |_0^T$$

$$= -\frac{Pb_-^2}{2c}(e^{-2cT} - 1) = \frac{Pb_-^2}{2c}(1 - e^{-2cT})$$

$$= \frac{PTb_-^2}{2cT}(1 - e^{-2cT})$$

3) $s_{ACI}^{wc}(t) = \sqrt{P}\, b \sum_{\substack{k=-M/2 \\ k \neq 0}}^{M/2} \frac{e^{j\omega_k t}}{1 + j\frac{\omega_k}{c}} \qquad 0 < t < T$

$$|s_{ACI}^{wc}(t)|^2 = P|b|^2 \sum_{\substack{k=-M/2 \\ k\neq 0}}^{M/2} \sum_{\substack{h=-M/2 \\ h\neq 0}}^{M/2} \frac{e^{j(\omega_k-\omega_h)t}}{(1+j\frac{\omega_k}{c})(1-j\frac{\omega_h}{c})} \qquad 0 < t < T$$

$$\int_0^T |s_{ACI}^{wc}(t)|^2 dt = P|b|^2 \sum_{\substack{k=-M/2 \\ k\neq 0}}^{M/2} \sum_{\substack{h=-M/2 \\ h\neq 0}}^{M/2} \frac{1}{(1+j\frac{\omega_k}{c})(1-j\frac{\omega_h}{c})} \int_0^T e^{j(\omega_k-\omega_h)t} dt$$

$$= P|b|^2 \sum_{\substack{k=-M/2 \\ k\neq 0}}^{M/2} \sum_{\substack{h=-M/2 \\ h\neq 0}}^{M/2} \frac{1}{(1+j\frac{\omega_k}{c})(1-j\frac{\omega_h}{c})} \frac{1}{j(\omega_k-\omega_h)} e^{j(\omega_k-\omega_h)t} \Big|_0^T$$

$$= P|b|^2 \sum_{\substack{k=-M/2 \\ k\neq 0}}^{M/2} \sum_{\substack{h=-M/2 \\ h\neq 0}}^{M/2} \frac{1}{(1+j\frac{\omega_k}{c})(1-j\frac{\omega_h}{c})} \frac{\frac{1}{c}}{j(\frac{\omega_k}{c}-\frac{\omega_h}{c})}$$

$$[e^{j(\frac{\omega_k}{c}-\frac{\omega_h}{c})cT} - 1$$

$$= \frac{PT|b|^2}{cT} \sum_{\substack{k=-M/2 \\ k\neq 0}}^{M/2} \sum_{\substack{h=-M/2 \\ h\neq 0}}^{M/2} \frac{e^{j(\frac{\omega_k}{c}-\frac{\omega_h}{c})cT}-1}{j(1+j\frac{\omega_k}{c})(1-j\frac{\omega_h}{c})(\frac{\omega_k}{c}-\frac{\omega_h}{c})}$$

$$\omega_k = \frac{2\pi kI}{T}, I = \text{INTEGER} > 0$$

THEN $\displaystyle\int_0^T e^{j(\omega_k-\omega_h)t} dt = \begin{cases} T, & \omega_k=\omega_h \\ 0, & \omega_k\neq\omega_h \end{cases}$

$$\int_0^T |s_{ACI}^{wc}(t)|^2 dt = PT|b|^2 \sum_{\substack{k=-M/2 \\ k\neq 0}}^{M/2} \frac{1}{1+(\frac{\omega_k}{c})^2}$$

4) $$2Re\{s_B(t)s_{ISI}^*(t)\} = 2Re\{\sqrt{P}\, b_{0,0}(1-e^{-ct})\sqrt{P}\, b_- e^{-ct}\}$$

$$= 2Pb_{0,0}\, b_-(e^{-ct}-e^{-2ct}) \quad 0<t<T$$

$$\int_0^T 2Re\{s_B(t)s_{ISI}^*(t)\}dt = 2Pb_{0,0}b_-\{\tfrac{e^{-ct}}{-c}\Big|_0^T - \tfrac{e^{-2ct}}{-c}\Big|_0^T\}$$

$$= 2Pb_{0,0}b_-\{\tfrac{e^{-cT}-1}{-c} + \tfrac{e^{-2cT}-1}{2c}$$

$$= \frac{PTb_{0,0}b_-}{cT}(1-2e^{-cT}+e^{-2cT})$$

5) $$2Re\{s_B^*(t)s_{ACI}(t)\} = 2Re\{\sqrt{P}\, b_{0,0}(1-e^{-ct})\sqrt{P}\, b \sum_{\substack{k=-M/2\\k\neq 0}}^{M/2} \frac{e^{j\omega_k t}}{1+j\frac{\omega_k}{c}}$$

$$= 2Pb_{0,0}Re\{b \sum_{\substack{k=-M/2\\k\neq 0}}^{M/2} \frac{e^{j\omega_k t}-e^{-(c-j\omega_k)t}}{1+j\frac{\omega_k}{c}}\}$$

$$\int_0^T 2Re\{s_B^*(t)s_{ACI}(t)\}dt = 2Pb_{0,0}Re\{b \sum_{\substack{k=-M/2\\k\neq 0}}^{M/2} \frac{1}{1+j\frac{\omega_k}{c}}$$

$$[\int_0^T e^{j\omega_k t}dt - \int_0^T e^{-(c-j\omega_k)}dt]\}$$

$\omega_k = \frac{2\pi kI}{T}$, I = INTEGER>0

$$\int_0^T e^{j\omega_k t}dt = \frac{1}{j\omega_k}(e^{j\omega_k t} - 1) = 0$$

$$\int_0^T e^{-(c-j\omega_k)t}dt = \frac{e^{-(c-j\omega_k)T}-1}{-(c-j\omega_k)} = \frac{1}{c}\frac{1-e^{-cT}e^{j\omega_k T}}{1-j\frac{\omega_k}{c}} = \frac{1}{c}\frac{1-e^{-cT}}{1-j\frac{\omega_k}{c}}$$

$$\int_0^T 2Re\{s_B^*(t)s_{ACI}(t)\}dt = 2Pb_{0,0}Re\{b\sum_{\substack{k=-M/2\\k\neq 0}}^{M/2}(\frac{1}{1+j\frac{\omega_k}{c}})(-\frac{1}{c}\frac{1-e^{-cT}}{1-j\frac{\omega_k}{c}})\}$$

$$= -\frac{2PTb_{0,0}Re\{b\}}{cT}(1-e^{-cT})\sum_{\substack{k=-M/2\\k\neq 0}}^{M/2}\frac{1}{1+(\frac{\omega_k}{c})^2}$$

6) $$2Re\{s_{ISI}^*(t)s_{ACI}(t)\} = 2Re\{\sqrt{P}\,b_-e^{-ct}\sqrt{P}\,b\sum_{\substack{k=-M/2\\k\neq 0}}^{M/2}\frac{e^{j\omega_k t}}{1+j\frac{\omega_k}{c}}$$

$$= 2Pb_-Re\{b\sum_{\substack{k=-M/2\\k\neq 0}}^{M/2}\frac{e^{-(c-j\omega_k)t}}{1+j\frac{\omega_k}{c}}\}$$

$$\int_0^T 2Re\{s_{ISI}^*(t)s_{ACI}(t)\} = 2Pb_-Re\{b\}\sum_{\substack{k=-M/2\\k\neq 0}}^{M/2}\frac{1}{c}\frac{1-e^{-cT}}{(1-j\frac{\omega_k}{c})(1+j\frac{\omega_k}{c})}$$

$$= \frac{2PTb_-Re\{b\}}{cT}(1-e^{-cT})\sum_{\substack{k=-M/2\\k\neq 0}}^{M/2}\frac{1}{1+(\frac{\omega_k}{c})^2}$$

SUMMARY OF WORST-CASE ANALYSIS

$$\omega_k = \frac{2\pi k l}{T} \qquad l = \text{INTEGER} > 0$$

$$X = \mathscr{A} \int_0^T |s(t)|^2 dt$$

$$= \mathscr{A}\{PTb_{0,0}^2[1 - \tfrac{2}{cT}(1 - e^{-cT}) + \tfrac{1}{2cT}(1 - e^{-2cT})] + \tfrac{PTb_-^2}{2cT}(1 - e^{-2cT})$$

$$+PT|b|^2 \sum_{\substack{k=-M/2 \\ k\neq 0}}^{M/2} \frac{1}{1+(\frac{\omega_k}{c})^2} + \frac{PTb_{0,0}b_-}{cT}(1 - 2e^{-cT} + e^{-2cT})$$

$$-\frac{2PTb_{0,0}Re\{b\}}{cT}(1 - e^{-cT}) \sum_{\substack{k=-M/2 \\ k\neq 0}}^{M/2} \frac{1}{1+(\frac{\omega_k}{c})^2}$$

$$+\frac{2PTb_-Re\{b\}}{cT}(1 - e^{-cT}) \sum_{\substack{k=-M/2 \\ k\neq 0}}^{M/2} \frac{1}{1+(\frac{\omega_k}{c})^2}\}$$

$$X = \mathscr{A}PT\{b_{0,0}^2[1 - \tfrac{2}{cT}(1 - e^{-cT}) + \tfrac{1}{2cT}(1 - e^{-2cT})]$$

$$+b_-^2[\tfrac{1}{2cT}(1 - e^{-2cT})] + b_{0,0}b_-[\tfrac{1}{cT}(1 - 2e^{-cT} + e^{-2cT})]$$

$$+[|b|^2 + \tfrac{2}{cT}(1 - e^{-cT})Re\{b\}(b_- - b_{0,0})] \sum_{\substack{k=-M/2 \\ k\neq 0}}^{M/2} \frac{1}{1+(\frac{2\pi k l}{cT})^2}\}$$

WHERE $b \in \{0, e^{j\phi}\}, Re\{b\} \in \{0, \cos\phi\}, |b|^2 \in \{0, 1\}$

EXACT ANALYSIS

$$1) \int_0^T |s_B(t)|^2 dt = PTb_{0,0}^2[1 - \tfrac{2}{cT}(1 - e^{-cT}) + \tfrac{1}{2cT}(1 - e^{-2cT})]$$

$$2) \; s_{ISI}(t) = \sqrt{P}\, e^{-ct} \sum_{i=-L_0}^{-1} b_{0,i}(e^{(i+1)cT} - e^{icT}) \qquad 0 < t < T$$

$$|s_{ISI}(t)|^2 = Pe^{-2ct}[\sum_{i=-L_0}^{-1} b_{0,i}(e^{(i+1)cT} - e^{icT})]^2 \qquad 0 < t < T$$

$$\int_0^T |s_{ISI}(t)|^2 dt = \tfrac{PT}{2cT}(1 - e^{-2cT})[\sum_{i=-L_0}^{-1} b_{0,i}(e^{(i+1)cT} - e^{icT})]^2$$

$$3) \; s_{ACI}(t) = \sqrt{P}\, ce^{-ct} \sum_{\substack{k=-M/2 \\ k \neq 0}}^{M/2} \frac{1}{c(1+j\frac{\omega_k}{c})}\{[\sum_{l=-L}^{-1} b_{k,l}(e^{(1+j\frac{\omega_k}{c})(l+1)cT}$$

$$-e^{(1+j\frac{\omega_k}{c})lcT})] + b_{k,0}(e^{(1+j\frac{\omega_k}{c})ct} - 1)\} \qquad 0 < t < T$$

$$s_{ACI}(t) = \sqrt{P}\, e^{-ct} \sum_{\substack{k=-M/2 \\ k\neq 0}}^{M/2} \sum_{l=-L}^{-1} \frac{1}{1+j\frac{\omega_k}{c}} b_{k,l} (e^{(1+j\frac{\omega_k}{c})(l+1)cT}$$

$$-e^{(1+j\frac{\omega_k}{c})lcT}) + \sqrt{P} \sum_{\substack{k=-M/2 \\ k\neq 0}}^{M/2} \frac{b_{k,0}}{1+j\frac{\omega_k}{c}} (e^{j\omega_k t} - e^{-ct})$$

$$0 < t < T$$

$$|s_{ACI}(t)|^2 = Pe^{-2ct} | \sum_{\substack{k=-M/2 \\ k\neq 0}}^{M/2} \sum_{l=-L}^{-1} \frac{b_{k,l}}{1+j\frac{\omega_k}{c}} b_{k,l} (e^{(1+j\frac{\omega_k}{c})(l+1)cT} - e^{(1+j\frac{\omega_k}{c})lcT} |^2$$

$$+P \sum_{\substack{k=-M/2 \\ k\neq 0}}^{M/2} \sum_{\substack{m=-M/2 \\ m\neq 0}}^{M/2} \frac{b_{k,0} b^*_{m,0}}{(1+j\frac{\omega_k}{c})(1-j\frac{\omega_m}{c})} (e^{j\omega_k t} - e^{-ct})(e^{-j\omega_m t} - e^{-ct})$$

$$+2P Re\{ \sum_{\substack{k=-M/2 \\ k\neq 0}}^{M/2} \sum_{\substack{m=-M/2 \\ m\neq 0}}^{M/2} \sum_{l=-L}^{-1} \frac{b_{k,l} b^*_{m,0}}{(1+j\frac{\omega_k}{c})(1-j\frac{\omega_m}{c})} (e^{(1+j\frac{\omega_k}{c})(l+1)cT}$$

$$-e^{(1+j\frac{\omega_k}{c})lcT})(e^{-(1+j\frac{\omega_m}{c})ct} - e^{-2ct})\}$$

EVALUATE $\int_0^T |s_{ACI}(t)|^2 dt$:

a) $\int_0^T |s_{ACI}(t)|_1^2 dt = \frac{PT}{2cT}(1 - e^{-2cT}) | \sum_{\substack{k=-M/2 \\ k\neq 0}}^{M/2} \sum_{l=-L}^{-1} \frac{b_{k,l}}{1+j\frac{\omega_k}{c}}$

$$(e^{(1+j\frac{\omega_k}{c})(l+1)cT} - e^{(1+j\frac{\omega_k}{c})lcT}) |^2$$

SPECIAL CASE : $\omega_k = \frac{2\pi k I}{T}$ I = INTEGER > 0

$$= \frac{PT}{2cT}(1 - e^{-2cT})\Big| \sum_{\substack{k=-M/2 \\ k\neq 0}}^{M/2} \sum_{l=-L}^{-1} \frac{b_{k,l}}{1+j\frac{2\pi k I}{cT}}(e^{(l+1)cT} - e^{lcT})\Big|^2$$

b) $$\int_0^T |s_{ACI}(t)|_2^2 dt = P \sum_{\substack{k=-M/2 \\ k\neq 0}}^{M/2} \sum_{\substack{m=-M/2 \\ m\neq 0}}^{M/2} \frac{b_{k,0}\, b^*_{m,0}}{(1+j\frac{\omega_k}{c})(1-j\frac{\omega_m}{c})}\Big\{\int_0^T e^{j(\omega_k-\omega_m)t}dt$$

$$-\int_0^T e^{-(c-j\omega_k)t}dt - \int_0^T e^{-(c+j\omega_m)t}dt + \int_0^T e^{-2ct}dt\Big\}$$

$$= P \sum_{\substack{k=-M/2 \\ k\neq 0}}^{M/2} \sum_{\substack{m=-M/2 \\ m\neq 0}}^{M/2} \frac{b_{k,0} b^*_{m,0}}{(1+j\frac{\omega_k}{c})(1-j\frac{\omega_m}{c})}\Big\{ \begin{cases} T, & \omega_k = \omega_m \\ \frac{e^{j(\omega_k-\omega_m)T}-1}{j(\omega_k-\omega_m)}, & \omega_k \neq \omega_m \end{cases}$$

$$+\frac{e^{-(c-j\omega_k)T}-1}{c-j\omega_k} + \frac{e^{-(c+j\omega_m)T}-1}{c+j\omega_m} + \frac{1}{2c}(1 - e^{-2cT})\Big\}$$

$$= \frac{PT}{cT} \sum_{\substack{k=-M/2 \\ k\neq 0}}^{M/2} \sum_{\substack{m=-M/2 \\ m\neq 0}}^{M/2} \frac{b_{k,0} b^*_{m,0}}{(1+j\frac{\omega_k}{c})(1-j\frac{\omega_m}{c})}\Big\{ \begin{cases} cT, & \omega_k = \omega_m \\ \frac{e^{j(\frac{\omega_k}{c}-\frac{\omega_m}{c})}-1}{j(\frac{\omega_k}{c}-\frac{\omega_m}{c})}, & \omega_k \neq \omega_m \end{cases}$$

$$+\frac{e^{-(1-j\frac{\omega_k}{c})cT}-1}{1-j\frac{\omega_k}{c}} + \frac{e^{-(1+j\frac{\omega_m}{c})cT}-1}{1+j\frac{\omega_m}{c}} + \frac{1}{2}(1 - e^{-2cT})\Big\}$$

SPECIAL CASE : $\omega_k = \frac{2\pi k I}{T}$ I = INTEGER > 0

$$\omega_m = \frac{2\pi m I}{T}$$

$$\int_0^T e^{j(\omega_k-\omega_m)t}dt = \begin{cases} T, & \omega_k=\omega_m \\ 0, & \omega_k\neq\omega_m \end{cases} = \begin{cases} \frac{cT}{c}, & \omega_k=\omega_m \\ \frac{0}{c}, & \omega_k\neq\omega_m \end{cases}$$

$$e^{j\omega_k T} = e^{-j\omega_k T} = 1$$

$$= PT\{\sum_{\substack{k=-M/2\\k\neq 0}}^{M/2} \frac{|b_{k,0}|^2}{1+(\frac{2\pi kl}{cT})^2} + \frac{1}{cT}\sum_{\substack{k=-M/2\\k\neq 0}}^{M/2}\sum_{\substack{m=-M/2\\m\neq 0}}^{M/2} \frac{b_{k,0}b^*_{m,0}}{(1+j\frac{2\pi kl}{cT})(1-j\frac{2\pi ml}{cT})}[\frac{e^{-cT}-1}{1-j\frac{2\pi kl}{cT}}$$

$$+\frac{e^{-cT}-1}{1+j\frac{2\pi ml}{cT}} + \frac{1}{2}(1-e^{-2cT})]\}$$

$$\text{C)} \int_0^T |s_{ACI}(t)|_3^2\,dt = 2PRe\{\sum_{\substack{k=-M/2\\k\neq 0}}^{M/2}\sum_{\substack{m=-M/2\\m\neq 0}}^{M/2}\sum_{l=-L}^{-1} \frac{b_{k,l}b^*_{m,0}}{(1+j\frac{\omega_k}{c})(1-j\frac{\omega_m}{c})}$$

$$(e^{(1+j\frac{\omega_k}{c})(l+1)cT} - e^{(1+j\frac{\omega_k}{c})lcT})[\int_0^T e^{-(1+j\frac{\omega_m}{c})ct}dt - \int_0^T e^{-2ct}dt]\}$$

$$= 2\frac{PT}{cT}Re\{\sum_{\substack{k=-M/2\\k\neq 0}}^{M/2}\sum_{\substack{m=-M/2\\m\neq 0}}^{M/2}\sum_{l=-L}^{-1} \frac{b_{k,l}b^*_{m,0}}{(1+j\frac{\omega_k}{c})(1-j\frac{\omega_m}{c})}$$

$$(e^{(1+j\frac{\omega_k}{c})(l+1)cT} - e^{(1+j\frac{\omega_k}{c})lcT})[\frac{1-e^{-(1+j\frac{\omega_m}{c})cT}}{1+j\frac{\omega_m}{c}} - \frac{1}{2}(1-e^{-2cT})]\}$$

SPECIAL CASE : $\omega_k = \frac{2\pi kl}{T}$ $\qquad l$ = INTEGER > 0

$$= 2\frac{PT}{cT} Re\{ \sum_{\substack{k=-M/2 \\ k\neq 0}}^{M/2} \sum_{\substack{m=-M/2 \\ m\neq 0}}^{M/2} \sum_{l=-L}^{-1} \frac{b_{k,l} b^*_{m,0}}{(1+j\frac{2\pi kl}{cT})(1-j\frac{2\pi ml}{cT})} (e^{(l+1)cT} - e^{lcT})[\frac{1-e^{-cT}}{1+j\frac{2\pi ml}{cT}}$$

$$-\frac{1}{2}(1 - e^{-2cT})]\}$$

$$\int_0^T |s_{ACI}\ t)|^2 dt = \frac{PT}{2cT}(1 - e^{-2cT})| \sum_{\substack{k=-M/2 \\ k\neq 0}}^{M/2} \sum_{l=-L}^{-1} \frac{b_{k,l}}{1+j\frac{\omega_k}{c}} (e^{(1+j\frac{\omega_k}{c})(l+1)cT}$$

$$-e^{(1+j\frac{\omega_k}{c})lcT})|^2 + \frac{PT}{CT} \sum_{\substack{k=-M/2 \\ k\neq 0}}^{M/2} \sum_{\substack{m=-M/2 \\ m\neq 0}}^{M/2} \frac{b_{k,0} b^*_{m,0}}{(1+j\frac{\omega_k}{c})(1-j\frac{\omega_m}{c})}$$

$$\{ \left\{ \begin{matrix} cT, & \omega_k = \omega_m \\ \frac{e^{j(\frac{\omega_k}{c}-\frac{\omega_m}{c})cT}-1}{j(\frac{\omega_k}{c}-\frac{\omega_m}{c})}, & \omega_k \neq \omega_m \end{matrix} \right\} + \frac{e^{-(1-j\frac{\omega_k}{c})cT}-1}{1-j\frac{\omega_k}{c}}$$

$$+\frac{e^{-(1+j\frac{\omega_m}{c})cT}-1}{1+j\frac{\omega_m}{c}} + \frac{1}{2}(1 - e^{-2cT})\}$$

$$+2\frac{PT}{cT} Re\{ \sum_{\substack{k=-M/2 \\ k\neq 0}}^{M/2} \sum_{\substack{m=-M/2 \\ m\neq 0}}^{M/2} \sum_{l=-L}^{-1} \frac{b_{k,l} b^*_{m,0}}{(1+j\frac{\omega_k}{c})(1-j\frac{\omega_m}{c})}$$

$$(e^{(1+j\frac{\omega_k}{c})(l+1)cT} - e^{(1+j\frac{\omega_k}{c})lcT})[\frac{1-e^{-(1+j\frac{\omega_m}{c})cT}}{1+j\frac{\omega_m}{c}}$$

$$-\frac{1}{2}(1 - e^{-2cT})]\}$$

SPECIAL CASE :

$$\int_0^T |s_{ACI}(t)|^2 dt = \frac{PT}{2cT}(1-e^{-2cT})|\sum_{\substack{k=-M/2\\k\neq0}}^{M/2}\sum_{l=-L}^{-1}\frac{b_{k,l}}{1+j\frac{2\pi kl}{cT}}(e^{(l+1)cT}-e^{lcT})|^2$$

$$+PT\{\sum_{\substack{k=-M/2\\k\neq0}}^{M/2}\frac{|b_{k,0}|^2}{1+(\frac{2\pi kl}{cT})^2}+\frac{1}{cT}\sum_{\substack{k=-M/2\\k\neq0}}^{M/2}\sum_{\substack{m=-M/2\\m\neq0}}^{M/2}\frac{b_{k,0}b^*_{m,0}}{(1+j\frac{2\pi kl}{cT})(1-j\frac{2\pi ml}{cT})}$$

$$[\frac{e^{-cT}-1}{1-j\frac{2\pi kl}{cT}}+\frac{e^{-cT}-1}{1+j\frac{2\pi ml}{cT}}+\frac{1}{2}(1-e^{-2cT})]\}$$

$$+2\frac{PT}{cT}Re\{\sum_{\substack{k=-M/2\\k\neq0}}^{M/2}\sum_{\substack{m=-M/2\\m\neq0}}^{M/2}\sum_{l=-L}^{-1}\frac{b_{k,l}b^*_{m,0}}{(1+j\frac{2\pi kl}{cT})(1-j\frac{2\pi ml}{cT})}(e^{(l+1)cT}$$

$$-e^{lcT})[\frac{1-e^{-cT}}{1+j\frac{2\pi ml}{cT}}-\frac{1}{2}(1-e^{-2cT})]\}$$

4) $\int_0^T 2Re\{s_B(t)s^*_{ISI}(t)\}dt$

$$=\int_0^T 2[\sqrt{P}\,b_{0,0}(1-e^{-ct})][\sqrt{P}\,e^{-ct}\sum_{i=-L_0}^{-1}b_{0,i}(e^{(i+1)cT}-e^{icT})]dt$$

$$=2Pb_{0,0}\sum_{i=-L_0}^{-1}b_{0,i}(e^{(i+1)cT}-e^{icT})\int_0^T(e^{-ct}-e^{-2ct})dt$$

$$\int_0^T 2Re\{s_B(t)s_{ISI}^*(t)\}dt$$

$$= 2Pb_{0,0} \sum_{i=-L_0}^{-1} b_{0,i}(e^{(i+1)cT} - e^{icT})[\tfrac{e^{-ct}}{-c}|_0^T - \tfrac{e^{-2ct}}{-2c}|_0^T]$$

$$= \frac{2PTb_{0,0}}{cT} \sum_{i=-L_0}^{-1} b_{0,i}(e^{(i+1)cT} - e^{icT})[1 - e^{-cT} + \tfrac{1}{2}e^{-2cT} - \tfrac{1}{2}]$$

$$= \frac{PTb_{0,0}}{cT} \sum_{i=-L_0}^{-1} b_{0,i}(e^{(i+1)cT} - e^{icT})(1 + e^{-2cT} - 2e^{-cT})$$

$$= \frac{PTb_{0,0}}{cT}(1 + e^{-2cT} - 2e^{-cT}) \sum_{i=-L_0}^{-1} b_{0,i}(e^{(i+1)cT} - e^{icT})$$

5) $\int_0^T 2Re\{s_B^*(t)s_{ACI}(t)\}dt$

$$= 2\int_0^T Re\{[\sqrt{P}\, b_{0,0}(1 - e^{-ct})]$$

$$[\sqrt{P}\, e^{-ct} \sum_{\substack{k=-M/2 \\ k\neq 0}}^{M/2} \sum_{l=-L}^{-1} \frac{b_{k,l}}{1+j\frac{\omega_k}{c}}(e^{(1+j\frac{\omega_k}{c})(l+1)cT} - e^{(1+j\frac{\omega_k}{c})lcT})$$

$$+\sqrt{P} \sum_{\substack{k=-M/2 \\ k\neq 0}}^{M/2} \frac{b_{k,0}}{1+j\frac{\omega_k}{c}}(e^{j\omega_k t} - e^{-ct})]\}dt$$

$$= 2\int_0^T Re\{Pb_{0,0}(1-e^{-ct})e^{-ct}$$

$$\sum_{\substack{k=-M/2\\k\neq 0}}^{M/2}\sum_{l=-L}^{-1}\frac{b_{k,l}}{1+j\frac{\omega_k}{c}}(e^{(1+j\frac{\omega_k}{c})(l+1)cT}-e^{(1+j\frac{\omega_k}{c})lcT})+Pb_{0,0}$$

$$\sum_{\substack{k=-M/2\\k\neq 0}}^{M/2}\frac{b_{k,0}}{1+j\frac{\omega_k}{c}}(e^{j\omega_k t}-e^{-ct}-e^{-(1-j\frac{\omega_k}{c})ct}+e^{-2ct})\}dt$$

$$= 2Pb_{0,0}Re\{\sum_{\substack{k=-M/2\\k\neq 0}}^{M/2}\sum_{l=-L}^{-1}\frac{b_{k,l}}{1+j\frac{\omega_k}{c}}(e^{(1+j\frac{\omega_k}{c})(l+1)cT}-e^{(1+j\frac{\omega_k}{c})lcT})$$

$$\int_0^T e^{-ct}-e^{-2ct})dt+\sum_{\substack{k=-M/2\\k\neq 0}}^{M/2}\frac{b_{k,0}}{1+j\frac{\omega_k}{c}}$$

$$\int_0^T(e^{j\omega_k t}-e^{-ct}-e^{-(1-j\frac{\omega_k}{c})ct}+e^{-2ct})dt\}$$

$$= 2Pb_{0,0}Re\{\frac{1}{c}(\frac{1}{2}+\frac{1}{2}e^{-2cT}-e^{-cT})\sum_{\substack{k=-M/2\\k\neq 0}}^{M/2}\sum_{l=-L}^{-1}\frac{b_{k,l}}{1+j\frac{\omega_k}{c}}(e^{(1+j\frac{\omega_k}{c})(l+1)cT}$$

$$-e^{(1+j\frac{\omega_k}{c})lcT})+\sum_{\substack{k=-M/2\\k\neq 0}}^{M/2}\frac{b_{k,0}}{1+j\frac{\omega_k}{c}}(\frac{e^{j\omega_k T}-1}{j\omega_k}+\frac{e^{-cT}-1}{c}-\frac{e^{-2cT}-1}{2c}+\frac{e^{-(1-j\frac{\omega_k}{c})cT}-1}{(1-j\frac{\omega_k}{c})c})\}$$

$$= \frac{PTb_{0,0}}{cT} Re\{(1+e^{-2cT}-2e^{-cT}) \sum_{\substack{k=-M/2 \\ k\neq 0}}^{M/2} \sum_{l=-L}^{-1} \frac{b_{k,l}}{1+j\frac{\omega_k}{c}} (e^{(1+j\frac{\omega_k}{c})(l+1)cT}$$

$$-e^{(1+j\frac{\omega_k}{c})lcT}) + 2 \sum_{\substack{k=-M/2 \\ k\neq 0}}^{M/2} \frac{b_{k,0}}{1+j\frac{\omega_k}{c}} (\frac{e^{j\omega_k T}-1}{j\frac{\omega_k}{c}} + e^{-cT} - 1 - \frac{1}{2}e^{-2cT} + \frac{1}{2}$$

$$+\frac{e^{-(1-j\frac{\omega_k}{c})cT}-1}{1-j\frac{\omega_k}{c}})\}$$

SPECIAL CASE : $\omega_k = \frac{2\pi kI}{T}$ $\qquad I$ = FIXED INTEGER > 0

$$\int_0^T e^{j\omega_k t} dt = \frac{1}{j\omega_k}(e^{j\omega_k T} - 1) = 0$$

THEN

$$\int_0^T 2Re\{s_B^*(t)s_{ACI}(t)\}dt$$

$$= \frac{PTb_{0,0}}{cT} Re\{(1+e^{-2cT}-2e^{-cT}) \sum_{\substack{k=-M/2 \\ k\neq 0}}^{M/2} \sum_{l=-L}^{-1} \frac{b_{k,l}}{1+j\frac{2\pi kI}{cT}} (e^{(1+j\frac{2\pi kI}{cT})(l+1)cT}$$

$$-e^{(1+j\frac{2\pi kI}{cT})lcT}) + 2 \sum_{\substack{k=-M/2 \\ k\neq 0}}^{M/2} \frac{b_{k,0}}{1+j\frac{2\pi kI}{cT}} (e^{-cT} - 1 - \frac{1}{2}e^{-2cT} + \frac{1}{2} + \frac{e^{-cT}-1}{1-j\frac{2\pi kI}{cT}})\}$$

6) $$\int_0^T 2Re\{s_{ISI}^*(t)s_{ACI}(t)\}dt$$

$$= 2Re\int_0^T [\sqrt{P}e^{-ct}\sum_{i=-L_0}^{-1} b_{0,i}(e^{(i+1)cT} - e^{icT})]$$

$$[\sqrt{P}e^{-ct}\sum_{\substack{k=-M/2\\k\neq 0}}^{M/2}\sum_{l=-L}^{-1}\frac{b_{k,l}}{1+j\frac{\omega_k}{c}}(e^{(1+j\frac{\omega_k}{c})(l+1)cT} - e^{(1+j\frac{\omega_k}{c})lcT})$$

$$+\sqrt{P}\sum_{\substack{k=-M/2\\k\neq 0}}^{M/2}\frac{b_{k,0}}{1+j\frac{\omega_k}{c}}(e^{j\omega_k t} - e^{-ct})]dt$$

$$= 2PRe\{\sum_{\substack{k=-M/2\\k\neq 0}}^{M/2}\sum_{l=-L}^{-1}\sum_{i=-L_0}^{-1}\frac{b_{0,i}b_{k,l}}{1+j\frac{\omega_k}{c}}(e^{(i+1)cT} - e^{icT})$$

$$(e^{(1+j\frac{\omega_k}{c})(l+1)cT} - e^{(1+j\frac{\omega_k}{c})lcT})\int_0^T e^{-2ct}dt + \sum_{\substack{k=-M/2\\k\neq 0}}^{M/2}\sum_{i=-L_0}^{-1}\frac{b_{0,i}b_{k,0}}{1+j\frac{\omega_k}{c}}$$

$$(e^{(i+1)cT} - e^{icT})\int_0^T (e^{-(1-j\frac{\omega_k}{c})ct} - e^{-2ct})dt\}$$

$$= 2PRe\{\sum_{\substack{k=-M/2\\k\neq 0}}^{M/2}\sum_{l=-L}^{-1}\sum_{i=-L_0}^{-1}\frac{b_{0,i}b_{k,l}}{1+j\frac{\omega_k}{c}}(e^{(i+1)cT} - e^{icT})$$

$$(e^{(1+j\frac{\omega_k}{c})(l+1)cT} - e^{(1+j\frac{\omega_k}{c})lcT})\frac{1}{2c}(1 - e^{-2cT})$$

$$+\sum_{\substack{k=-M/2\\k\neq 0}}^{M/2}\sum_{i=-L_0}^{-1}\frac{b_{0,i}b_{k,0}}{1+j\frac{\omega_k}{c}}(e^{(i+1)cT}-e^{icT})[\frac{1-e^{-(1-j\frac{\omega_k}{c})cT}}{(1-j\frac{\omega_k}{c})c}+\frac{1}{2c}(e^{-2cT}-1)]\}$$

$$=\frac{PT}{cT}\sum_{i=-L_0}^{-1}b_{0,i}(e^{(i+1)cT}-e^{icT})Re\{\sum_{\substack{k=-M/2\\k\neq 0}}^{M/2}\sum_{l=-L}^{-1}\frac{b_{k,l}}{1+j\frac{\omega_k}{c}}(1-e^{-2cT})$$

$$(e^{(1+j\frac{\omega_k}{c})(l+1)cT}-e^{(1+j\frac{\omega_k}{c})lcT})+\sum_{\substack{k=-M/2\\k\neq 0}}^{M/2}\frac{b_{k,0}}{1+j\frac{\omega_k}{c}}(\frac{2(1-e^{-(1-j\frac{\omega_k}{c})cT})}{1-j\frac{\omega_k}{c}}+e^{-2cT}-1)\}$$

SPECIAL CASE : $\omega_k=\frac{2\pi kI}{T}$ $\qquad I=$ INTEGER >0

$$\int_0^T 2Re\{s^*_{ISI}(t)s_{ACI}(t)dt$$

$$=\frac{PT}{cT}\sum_{i=-L_0}^{-1}b_{0,i}(e^{(i+1)cT}-e^{icT})Re\{\sum_{\substack{k=-M/2\\k\neq 0}}^{M/2}\sum_{l=-L}^{-1}\frac{b_{k,l}}{1+j\frac{2\pi kI}{cT}}(1-e^{-2cT})$$

$$(e^{(1+j\frac{2\pi kI}{cT})(l+1)cT}-e^{(1+j\frac{2\pi kI}{cT})lcT})+\sum_{\substack{k=-M/2\\k\neq 0}}^{M/2}\frac{b_{k,0}}{1+j\frac{2\pi kI}{cT}}[(\frac{2-2e^{-cT}}{1-j\frac{2\pi kI}{cT}})+e^{-2cT}-1]$$

WHERE $e^{(1+j\frac{\omega_k}{c})(l+1)cT}=e^{(c+j\omega_k)(l+1)T}=e^{(cT+j\omega_kT)(l+1)}$

$$=e^{cT(l+1)}e^{j\omega_kT(l+1)}=e^{cT(l+1)}$$

SIMILARLY $\Rightarrow e^{(1+j\frac{\omega_k}{c})lcT} = e^{(c+j\omega_k)lT} = e^{clT}$

$$\Rightarrow \int_0^T 2Re\{s_{ISI}^*(t)s_{ACI}(t)\}dt$$

$$= \frac{PT}{cT} \sum_{i=-L_0}^{-1} b_{0,i}(e^{(i+1)cT} - e^{icT})Re\{ \sum_{\substack{k=-M/2 \\ k\neq 0}}^{M/2} \sum_{l=-L}^{-1} \frac{b_{k,l}}{1+j\frac{2\pi kl}{cT}}$$

$$(1-e^{-2cT})(e^{cT(l+1)} - e^{cTl}) + \sum_{\substack{k=-M/2 \\ k\neq 0}}^{M/2} \frac{b_{k,0}}{1+j\frac{2\pi kl}{cT}}[(\frac{2-2e^{-cT}}{1-j\frac{2\pi kl}{cT}}) + e^{-2cT} - 1]\}$$

APPENDIX B

MATLAB COMPUTER PROGRAMS

exact analysis with optimal threshold signal out of the integrator, the signal X contains desired signal and ACI & ISI and postdetect noise ,The formula for the BER is :

$$\frac{1}{2^{M(L+1)+L_0}} \sum_{2^M \, pattern} p(b)$$

$$where \quad p(b) = \frac{1}{2}\left(\frac{1}{2\pi}\right)^M \Big\{ \underbrace{\int \cdots \int_0^{2\pi}}_{M} Q\left(\frac{\alpha - X_0(\Phi_{-M/2} \ldots \Phi_{M/2})}{\sqrt{N_0 T}}\right) d\Phi_{-M/2} \ldots d\Phi_{M/2}$$

$$+ \underbrace{\int \cdots \int_0^{2\pi}}_{M} Q\left(\frac{X_1(\Phi_{-M/2} \ldots \Phi_{M/2}) - \alpha}{\sqrt{N_0 T}}\right) d\Phi_{-M/2} \ldots d\Phi_{M/2} \Big\}$$

M is number of adjacent channels. Since we assume upper and lower channels are synchronized individually, i.e. for our model the power of $(1/2\pi)$ is fixed to 2, also we only have two integrations and two arguments.

```
M=4; k=[-M/2:-1 1:M/2];
 m=k;
% produce the controlled matrix b  to control 64 different bit patterns
m1=[ zeros(1,32) ones(1,32) ];
m2=[ zeros(1,16) ones(1,16) zeros(1,16) ones(1,16)];
m3=[];
  for i=1:4
     m3=[m3 [zeros(1,8) ones(1,8)]];
  end
m4=[];
  for i=1:8
     m4=[m4 [zeros(1,4) ones(1,4)]];
  end
m5=zeros(1,64);
 m6=[];
  for i=1:16
     m6=[m6 [zeros(1,2) ones(1,2)]];
  end
m7=[];
  for i=1:32
```

```
    m7=[m7 zeros(1,1) ones(1,1)];
  end

b=[m1;m2;m3;m4;m5;m6;m7]';

% signal to noise ratio range in dB
RPTN_DB=[10:0.5:25]; %RPTN_dB
ppp=10.^(0.1*RPTN_DB);
len1=length(ppp);

 % solalph function is provided by Professor Randy L. Borchardt

 n=10;
 [bpx,wfx]=grule(n); %bpx=bpy ,wfx=wfy

% single channel
 BER0=0.5*erfc(ppp/8^0.5);

pp=[]; thresh=[];   %thresh is not normalized threshold

for CT=[5 10 15 20]   % cT is Fabry-Perot filter parameter
       if CT==5
         qqq=[2:12 17 20];
       elseif CT==10
         qqq=[3:12 17 20];
       elseif CT==15
         qqq=[4:12 17 20];
       elseif CT==20
         qqq=[5:12 17 20];
       end

pe=[]; thresh1=[];

       for I=qqq
          BER=[]; thresh2=[];

         for RPTN=ppp
            ap=linspace(0,182,11);       %approximated thresholds
             x3=0;                       % first few loops to find out the threshold which make the
                                          % BER minimun don't care about the scale
           for i=1:64   x3=x3+...
              solalph('xx00',0,2*pi,2,bpx,wfx,0,2*pi,2,bpx,wfx,ap,b,CT,i,I,k,m,RPTN)+...
              solalph('xx11',0,2*pi,2,bpx,wfx,0,2*pi,2,bpx,wfx,ap,b,CT,i,I,k,m,RPTN);
           end
          [val,ind]=min(x3);
```

```
lef=ap(ind)-16.6;

  if lef<0 ,lef=0; end      % to aviod the threshold go beyond the negative side

  ap=linspace(lef,ap(ind)+16.6,11);
% two more time to find alpha
  x3=0;

 for i=1:64    x3=x3+...
     solalph('xx00',0,2*pi,2,bpx,wfx,0,2*pi,2,bpx,wfx,ap,b,CT,i,I,k,m,RPTN)+...
     solalph('xx11',0,2*pi,2,bpx,wfx,0,2*pi,2,bpx,wfx,ap,b,CT,i,I,k,m,RPTN);
 end

[val,ind]=min(x3);
lef=ap(ind)-3.1;

if lef<0 ,lef=0; end

ap=linspace(lef,ap(ind)+3.1,11);
% three more time to find alpha
 x3=0;

 for i=1:64
      x3=x3+...
    solalph('xx00',0,2*pi,2,bpx,wfx,0,2*pi,2,bpx,wfx,ap,b,CT,i,I,k,m,RPTN)+...
    solalph('xx11',0,2*pi,2,bpx,wfx,0,2*pi,2,bpx,wfx,ap,b,CT,i,I,k,m,RPTN);
 end

 [val,ind]=min(x3);
 lef=ap(ind)-0.57;

 if lef<0 ,lef=0; end

ap=linspace(lef,ap(ind)+0.57,8);
% four more time to find alpha
 x3=0;

for i=1:64
   x3=x3+...
   solalph('xx00',0,2*pi,2,bpx,wfx,0,2*pi,2,bpx,wfx,ap,b,CT,i,I,k,m,RPTN)+...
   solalph('xx11',0,2*pi,2,bpx,wfx,0,2*pi,2,bpx,wfx,ap,b,CT,i,I,k,m,RPTN);
 end

[val,ind]=min(x3);
ap=ap(ind);
```

```
    % after find optimal alpha use double integration
       qq3=0;

       for i=1:64
          qq3=qq3+( dbgquadm('xx00',0,2*pi,2,bpx,wfx,0,2*pi,2,...
                bpx,wfx,ap,b,CT,i,I,k,m,RPTN)+...
           dbgquadm('xx11',0,2*pi,2,bpx,wfx,0,2*pi,2,...
           bpx,wfx,ap,b,CT,i,I,k,m,RPTN) )/(1024*pi^2);
       end

       BER=[BER qq3]; thresh2=[thresh2 ap];

       if qq3<10^(-15)       %for save time only interesting in 10^(-15)
         ii=find(ppp==RPTN);
        BER(ii+1:len1)=5*10^(-116)*ones( 1,length(ii+1:len1) );
        thresh2(ii+1:len1)=(ap+2)*ones(1,length(ii+1:len1) );
        break
       end
    end

    pe=[pe;BER]; thresh1=[thresh1;thresh2];
  end

  pp=[pp;pe]; thresh=[thresh;thresh1];
end

time2=toc;
```

```
% worse case form appendix setting optimal threshold equal (x0+x1)/2
M=4;
I=linspace(0,6,101);           % set I value in linspceinteger to find minimum I
RPTN_DB=0:0.2:20;              % x-axis dB range
RPTN=10.^(0.1*RPTN_DB);  % change to ratio
% single channel
BER0=0.5*erfc(RPTN/8^0.5);
%find optimal alpha
pp=[];
  for CT=[5 10 15 20]
      x3=0;
      for k=1:0.5*M
         x3=x3+2./(1+(2*pi*k*I/CT).^2);
      end
     x0=(1-exp(-2*CT))/(2*CT)+x3*(1+2*(1-exp(-CT))/CT);
     x1=1-2*(1-exp(-CT))/CT+(1-exp(-2*CT))/(2*CT);
     [val,ind]=min(abs(x0-x1));
     I=ceil(I(ind));
     I=(I(ind));        % our minimum I value

    % set different I and to find the BER
      ss=I+1:I+10;
      BER=[];
     for I=ss
       x3=0;
         for k=1:0.5*M
            x3=x3+2/(1+(2*pi*k*I/CT)^2);
         end
         x0=(1-exp(-2*CT))/(2*CT)+x3*(1+2*(1-exp(-CT))/CT);
         x1=1-2*(1-exp(-CT))/CT+(1-exp(-2*CT))/(2*CT);
         BER=[BER;0.5*erfc( RPTN*(x1-x0)/8^0.5 ) ];
     end
         pp=[pp;BER];
  end

  semilogy(RPTN_DB,BER0,'--',RPTN_DB,pp(:,:))

  axis([10 19 10^(-15) 1])
```

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